CALCULUS AND DIFFERENTIAL EQUATIONS

MATH 1B

Lecture 33: Phase space analysis

Systems

33.1. The algorithm to analyze a phase-portrait of a system $x' = f(x,y), y' = g(x,y)$ of two differential equations consists of the steps: 1. **Factor** $f(x,y)$ and $g(x,y)$, 2. **Identify x-nullclines** $f(x,y) = 0$ and the **identify y-nullclines** $g(x,y) = 0$, 3. Find the **equilibria** 4. **Orient** the x-nullclines and y nullclines. 5. **Complete** the picture.

33.2. Here are the steps in the case of the Murray system:

$$\begin{aligned} x'(t) &= x(6-2x) - xy \\ y'(t) &= y(4-y) - xy \end{aligned}$$

1) Factor:

$$\begin{aligned} x'(t) &= x(6-2x-y) \\ y'(t) &= y(4-y-x) \end{aligned}$$

2) Nullclines: Draw the x-nullclines and y-nullclines with different colors. 3) Equilibria: The equilibrium points are where x-nullclines and y-nullclines intersect.

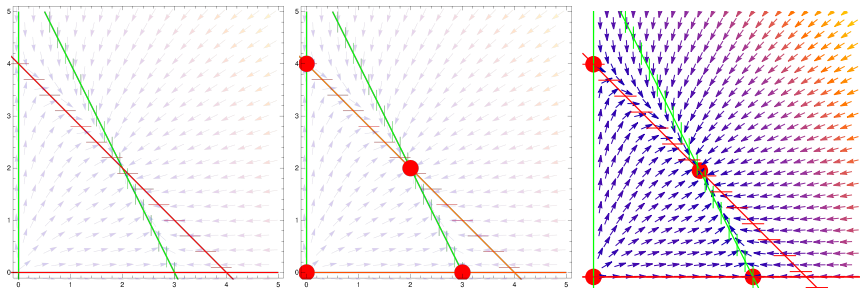

Figure 1. Phase space analysis.

4) Orient Nullclines: Orient the motion on the null-clines. 5) Complete the picture by drawing directions between nullclines:

33.3. We have now a complete picture and know that if we start with a population initial condition $x > 0$ and $y > 0$, then we end up at the equilibrium point $(2,2)$.

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