

Even if we had the possibility to find the anti-derivative of any function, for nonlinear systems this would not help. One can use numerical methods like the Euler method or then make Taylor expansions and find formulas for the Taylor coefficients but chaos theory has shown that one can also not find in general formulas for Taylor coefficients which converge.

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DSolve[{x'[t]== x[t]^2*y[t]^4 - x[t]^2,
        y'[t]==-x[t]^4 + x[t]*y[t]},{x[t],y[t]}, t]
```

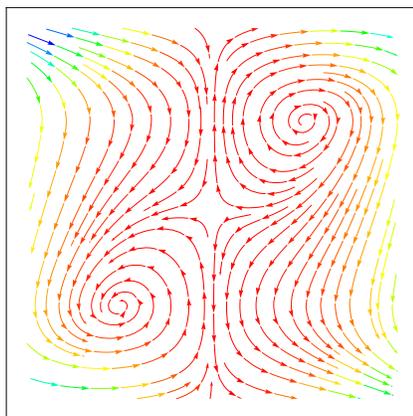


FIGURE 2. A random polynomial system

INTEGRABLE CASES

34.4. Keep in mind the **Harmonic oscillator** system $x' = y, y' = -c^2x$ which is equivalent to the spring systems $x'' = -c^2x$. It can be solved also because of the conserved energy $x^2 + y^2/c^2$. The conservation means that the ellipses $x^2 + y^2/c^2$ are invariant. There are many more systems like that. Any system $x'' = -V'(x)$ can be written as $x' = y, y' = -V'(x)$. It has the conserved energy quantity $y^2/2 + V(x)$. If we take the derivative, we get $yy' + V'(x)x' = -yV'(x)/2 + V'(x)y = 0$.

34.5. An example is the pendulum $x' = y, y' = -\sin(x)$ which is equivalent to $x'' = -\sin(x)$. It has the conserved quantity $y^2/2 + \cos(x)$.

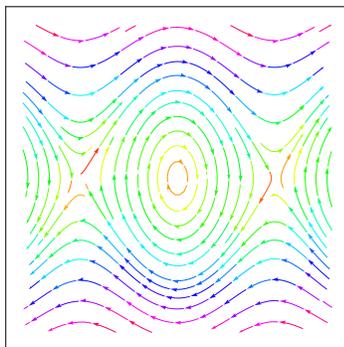


FIGURE 3. The pendulum