

DIFFERENTIAL GEOMETRY

MATH 136

Midterm Quiz

- 1) If $r : \mathbb{R}^2 \rightarrow \mathbb{R}^3$ then dr is called the of the r .
- 2) If $r : \mathbb{R}^3 \rightarrow \mathbb{R}^4$, then dr is a \times matrix.
- 3) If $r : \mathbb{R}^1 \rightarrow \mathbb{R}^3$ then dr is called the .
- 4) If $r : \mathbb{R}^2 \rightarrow \mathbb{R}^3$ parametrizes a surface, then r_u and r_v are to the surface.
- 5) If $r(t)$ parametrizes a curve, then $|r'(t)| = \sqrt{\det(g)}$ is called the .
- 6) The integral $\int_a^b \det(g) dt$ for a curve $r(t)$ was called integral.
- 7) If $r(u, v)$ parametrizes a surface, then $|r_u \times r_v|^2$ can be expressed using g as .
- 8) If r is a parametrization of a surface with $\det(g) \neq 0$ then the surface is called .
- 9) The result that the shortest connection between two points in the plane is a line is the theorem.
- 10) The curve with name has constant curvature and constant torsion.
- 11) A planar circle $[3 \cos(t), 3 \sin(t), 0]$ has curvature and torsion .
- 12) Every curve of constant curvature and torsion is congruent to a circle. True or False?

- 13) The fundamental form is defined as $g = dr^T dr$.
- 14) The fundamental form is defined as $h = -dr^T dn$.
- 15) The fundamental form is defined as $e = dn^T dn$.
- 16) The shape operator matrix A is defined using I and II as .
- 17) The curvature of a curve in the plane can become negative. True or False?
- 18) The curvature of a curve in space can become negative. True or False?
- 19) For a curve in \mathbb{R}^n , there are $n - 1$ curvatures $\kappa_1, \dots, \kappa_{n-1}$. The last one is called
- 20) The Hopf Umlaufsatz tells that for regular and curves, the rotation index is ± 1 .
- 21) The four vertex theorem holds also for non-simple curves. True or False?
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- 22) A vertex is a point, where the of the curvature is zero.
- 23) The evolute of a curve is $r(t) + N(t)/\kappa(t)$, where $N(t)$ is the .
- 24) The tennis ball theorem deals with curves on S^2 splitting it into two regions of .
- 25) The number $\iint_R \sqrt{\det(g)} \, dudv$ attached to a surface is called .
- 26) A curve is called if $dr^T dr$ is never zero.
- 27) When proving the existence theorem for differential equations, one used the fixed point theorem.
- 28) A surface is called if $dr^T dr$ has rank 2 everywhere.
- 29) The process was needed to produce an analog of the Frenet 3-frame.
- 30) The condition $e_k(t) \cdot e_k(t) = 1$ for all t implies $e'_k \cdot e_k =$.

- 31) $SU(2)$ is known as the special group.
- 32) The matrix entries of $SO(3)$ can not be larger than 1. True or False?
- 33) $SU(2)$ is important in physics because it is related to the force.
- 34) $SU(\text{})$ is the three dimensional sphere.
- 35) $so(5)$ is a space. True or False?
- 36) $su(5)$ is closed under addition. True or false?
- 37) The space $SO(2)$ is also known as the .
- 38) The fundamental theorem of linear algebra tells $\ker(A^T) = \text{}$.
- 39) The fundamental theorem of curves in \mathbb{R}^3 tells that and determine the curve up to congruence.
- 40) The second fundamental form II is defined using dr and dn as .
- 41) The second fundamental form II is symmetric. True or False?
- 42) The third fundamental form III is symmetric. True or False?
- 43) The normal map n is also called map.
- 44) The shape operator A is of a surface is a matrix.
- 45) The shape operator A is symmetric with respect to the standard dot product. True or False?
- 46) The shape operator A is symmetric with respect to the inner product given by g . True or False?
- 47) The Euler characteristic of a discrete 2-manifold is defined as .
- 48) The Euler characteristic of a 2-dimensional sphere is .
- 49) The Euler characteristic of a 2-dimensional torus is .

- 50) The Euler characteristic of a Projective plane is .
- 51) The Euler characteristic of a torus is .
- 52) Which blind mathematician first looked at discrete curvature? .
- 53) The curvature of a discrete 2-manifold is given as .
- 54) The discrete Gauss-Bonnet theorem tells that $\sum_{v \in V} K(v)$.
- 55) The Euler characteristic of a K_3 is .
- 56) III is determined by I and II. True or False? .
- 57) If r parametrizes $x^2+2y^2+3z^2 = 1$, then $\iint |n_u \times n_v| dudv$.
- 58) The discrete 2-sphere with 6 vertices is also called a .
- 59) There are two discrete 2-manifolds with constant curvature 0. The Klein bottle and the .
- 60) For graphs without triangles, where $\chi(G) = |V| - |E|$, the curvature is $K(v) =$.