PROBABILITY THEORY

MATH 154

Homework 1

Probability

Problem 1.1: a) You pick a random point (x, y) in the square $[-1, 1] \times [-1, 1]$. What is the probability that $x^2 + y^2 \leq 1$?
b) You pick a random point (x, y, z) in the unit cube $[-1, 1]^3$. What is the probability that $x^2 + y^2 + z^2 \leq 1$?
c) What is the probability that $x_1^2 + x_2^2 + ... + x_{1000}^2 \leq 1$ if the point $x = (x_1, \ldots, x_{1000})$ is chosen randomly in the 1000-dimensional unit cube $[-1, 1]^{1000}$.

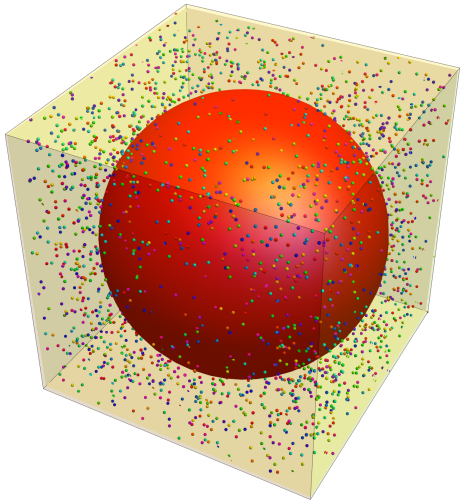

Figure 1. What is the probability to hit the sphere?

Problem 1.2: The card game "set" contains $81 = 3^4$ cards. Each card has one of 3 colors, one of 3 numbers, one of 3 shapes and one of 3 shades. It so models so the 4-dimensional vector space $\mathbb{Z}_3^4$ which is also called the field $GF(81)$. A collection of three cards is called a "set", if in each of the 3 categories, all three properties either agree or are all different. You randomly pick 3 cards from the 81. What is the probability to draw a set?

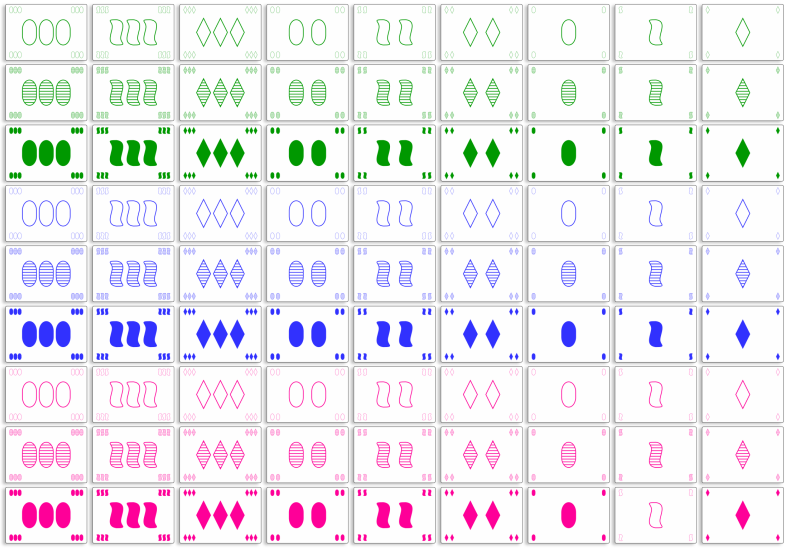

FIGURE 2. The game of set visualizes a 4 dimensional vector space

Problem 1.3: The probability density of a positive integer smaller than n is prime is about $1/\log(n)$ by the prime number theorem. What do you expect is the expected number of **prime twins** smaller than n?

Problem 1.4: a) Alex has three kids, and one of them is a girl. What is the probability that Alex has three girls?
b) Alex has three kids of different age and the oldest is a girl. What is the probability that Alex has three girls?

Problem 1.5: There are three boxes: a box containing two gold coins, a box containing two silver coins, and a box containing one gold coin and one silver coin. The three boxes are shuffled. You pick one box and pick a random coin from it. You notice it to be gold. What is the probability that the other coin from the same box is gold?