Lecture 36: Final checklist

Please see lecture 21 handout for the topics before the midterm.

Probability theory

- **Discrete random variable** $P[X = x_k] = p_k$ discrete set of values.
- **Continuous random variable** $P[X \in [a,b]] = \int_a^b f(x)\,dx$
- **Properties of random variables** $\mathrm{Var}, \mathrm{E}, \mathrm{Cov}, \sigma$.
- **Discrete distributions** Uniform, Binomial, Poisson
- **Continuous distributions** Uniform, Normal, Exponential
- **Chebychev's theorem** $P[|X - \mathrm{E}[X]| \geq c] \leq \frac{\mathrm{Var}[X]}{c^2}$.
- **Random walk** $S_n = \sum_{k=1}^n X_k$.
- **Law of large numbers** $S_n/n \to \mathrm{E}[X]$.
- **Normalized random variable** $X^* = (X - \mathrm{E}[X])/\sigma[X]$.
- **Convergence in distribution** $\mathrm{E}[\cos(tX_n)] \to \mathrm{E}[\cos(tX)]$ and $\mathrm{E}[\sin(tX_n)] \to \mathrm{E}[\sin(tX)]$ for all t.
- **Characteristic function** $\phi_X(t) = \mathrm{E}[e^{itX}]$.
- **Central limit theorem** $S_n^* \to N(0,1)$ in distribution if X_k are IID random variables with $\mathrm{E}[X^3] < \infty$.
- **Statistical inference** $\frac{S_n}{n} - \mathrm{E}[X] \sim N(0,1)\frac{\sigma}{\sqrt{n}}$.
- **Standard error** of $X_1, ..., X_n$ is $\sigma/\sqrt{n}$.
- **Stochastic vector** Nonnegative entries which add up to 1.
- **Markov Matrix = Stochastic Matrix** All columns are stochastic vectors.
- **Perron Frobenius theorem** Markov matrices have an eigenvalue 1. All other eigenvalue are smaller or equal than 1.
- **Positive matrix**. There is a unique largest eigenvalue 1.

Linear algebra

- **Determinant** $\sum_\pi (-1)^{|\pi|} A_{1\pi(1)} A_{2\pi(2)} \cdots A_{n\pi(n)}$
- **Laplace expansion** $\det(A) = (-1)^{1+1} A_{11} \det(B_{i1}) + \cdots + (-1)^{1+n} A_{n1} \det(B_{n1})$
- **Linearity of determinants** $\det(A)$ is linear in each row.
- **Determinant after row reduction** $\det(A) = \frac{(-1)^m}{c_1 \cdots c_k} \det(rref(A))$.
- **Determinant and invertibility** $\det(A \cdot B) = \det(A)\det(B)$.
- **Determinant and volume** $|\det(A)|$ is the volume of the parallelepiped spanned by the columns.
- **Cramer's rule** A_i replaces column by b, then $x_i = \frac{\det(A_i)}{\det(A)}$ solves $Ax = b$.
- **Adjugate** A_{ij} delete row i and column j. Call $B_{ij} = (-1)^{i+j} \det(A_{ji})$ the classical adjoint
- **Inverse** $[A^{-1}]_{ij} = (-1)^{i+j} \frac{\det(A_{ji})}{\det(A)}$
- **Eigenvectors and eigenvalues** $Av = \lambda v$
- **Characteristic polynomial** $f_A(\lambda) = \det(A - \lambda I_n)$
- **Eigenvalues** Roots or characteristic polynomial.
- **Eigenspace** kernel of $A - \lambda I_n$.
- **Geometric multiplicity of** λ dimension of eigenspace of λ.
- **Algebraic multiplicity of** λ_k m if $f_A(\lambda) = (\lambda - \lambda_k)^m g(\lambda)$ and $g(\lambda_k) \neq 0$.
- **Trace of matrix** Sum $\lambda_1 + \cdots + \lambda_n$ of eigenvalues.
- **Determinant of matrix** Product $\lambda_1 \lambda_2 \cdots \lambda_n$ of eigenvalues.
- **Discrete dynamical system** orbit $A^m x$
- **Closed form solution** $x = c_1 v_1 + \cdots + c_n v_n$, $A^m x = c_1 \lambda_1^n v_1 + \cdots + c_n \lambda_n^m v_n$.
- **Similar matrices** $S^{-1}AS$.
- **Diagonalizable matrix** There is an orthogonal eigenbasis.
- **Simple spectrum** All eigenvalues are different. Matrices with simple spectrum are diagonalizable.
- **Symmetric matrices** $A^T = A$. Symmetric matrices are diagonalizable.
- **Google matrix** $G = dA + (1-d)E$, where $E_{ij} = 1/n$, A is the adjusted adjacency matrix and d is a damping factor.