

Lecture 22: Distributions

The **geometric distribution** on $\Omega = \{0, 1, 2, 3, \dots\}$ is given by

$$P[X = k] = p(1 - p)^k .$$

An example is

1 Verify that this is a probability distribution and that

$$\sum_{k=0}^{\infty} p(1 - p)^k = 1 .$$

For example, if we throw a dice and wait until the first time 6 appears and X is the random variable which gives the number of times we failed, then

$$P[X = k] = (1 - p)^k p .$$

Here is the computation of the expectation:

2 Find the expectation

Solution: To find the expectation

$$\sum_{k=0}^{\infty} kp(1 - p)^k ,$$

define $q = 1 - p$ and look at the identity

$$\sum_{k=0}^{\infty} q^k = 1/(1 - q) .$$

Differentiation with respect to q gives

$$\sum_{k=0}^{\infty} kq^{k-1} = 1/(1 - q)^2$$

so that

$$\sum_{k=0}^{\infty} kq^k = q/(1 - q)^2 = q/p^2 .$$

Therefore,

$$\sum_{k=0}^{\infty} qpq^k = (1 - p)/p .$$