

Lecture 30: Diagonalization

To see whether matrices are similar it is useful to check that the eigenvalues are the same. Also the trace

$$\operatorname{tr}(A) = \lambda_1 + \dots + \lambda_n$$

and the determinant

$$\det(A) = \lambda_1 \cdot \dots \cdot \lambda_n$$

are the same.

It is also good to know that if all eigenvalues are different, then the matrix can be diagonalized.

If two eigenvalues are the same, it is good to compare the geometric multiplicity of the eigenvalues.

Determine from each of the following matrices whether they are similar or not:

$$1 \quad A = \begin{bmatrix} 2 & 1 \\ 0 & 3 \end{bmatrix} \quad B = \begin{bmatrix} 3 & 1 \\ 0 & 2 \end{bmatrix}$$

$$2 \quad A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} \quad B = \begin{bmatrix} 3 & 4 \\ 1 & 1 \end{bmatrix}$$

$$3 \quad A = \begin{bmatrix} 2 & 1 \\ 0 & 2 \end{bmatrix} \quad B = \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}$$

$$4 \quad A = \begin{bmatrix} 2 & 1 \\ 0 & 3 \end{bmatrix} \quad B = \begin{bmatrix} 2 & 0 \\ 0 & 3 \end{bmatrix}$$

$$5 \quad A = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix} \quad B = \begin{bmatrix} 2 & 1 \\ 1 & 0 \end{bmatrix}$$