Lecture 33: Markov matrices

1   The Land of Oz is blessed with three kinds of whether R (rain), N (nice weather) and S (snow). If they have a nice day, they are just as likely to have snow as rain the next day. If they have snow or rain, they have an even chance of having the same the next day. If there is change from snow or rain, only half of the time is this a change to a nice day. This is encoded by the following Markov chain:

$$A = \begin{bmatrix} 1/2 & 1/2 & 1/4 \\ 1/4 & 0 & 1/4 \\ 1/4 & 1/2 & 1/2 \end{bmatrix} .$$

Find the eigenvalues of A. Look at the trace and the determinant and use that one eigenvalue is 1.

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2   The following Ehrenfest model is explain diffusion of gases. Two urns contain four balls in total. At each step, one of the four balls is chosen at random and moved from the urn that it is in into the other urn. At the beginning all balls are in the first urn. The transition matrix is

$$P = \begin{bmatrix} 0 & 1/4 & 0 & 0 & 0 \\ 1 & 0 & 1/2 & 0 & 0 \\ 0 & 3/4 & 0 & 3/4 & 0 \\ 0 & 0 & 1/2 & 0 & 1 \\ 0 & 0 & 0 & 1/4 & 0 \end{bmatrix} .$$

You know that

$$\begin{bmatrix} 1 \\ 1 \\ -1 \\ -1 \\ 1 \end{bmatrix} , \begin{bmatrix} -2 \\ 2 \\ 1 \\ -1 \\ 0 \end{bmatrix} , \begin{bmatrix} -2 \\ 2 \\ -1 \\ 1 \\ 0 \end{bmatrix}$$

are eigenvectors. Can you find the eigenvalues?

2

Solution:
The eigenvalues are $-1, 1, -1/2, 1/2, 0$.

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[1]G. Kemeny, J. L. Snell, G. L. Thompson, Introduction to Finite Mathematics, 3rd ed. (Englewood Cliffs, NJ: Prentice-Hall, 1974).

[2]P. and T. Ehrenfest, Über zwei bekannte Einwände gegen das Boltzmannsche H-Theorem," Physikalishce Zeitschrift, vol. 8 (1907), pp. 311-314.