

Lecture 35: Spectral theorem

1 What can you say about the eigenvalues of the matrix

$$A = \begin{bmatrix} 2 & 1 & 1 & 1 & 2 \\ 1 & 4 & 1 & 1 & 2 \\ 1 & 1 & 4 & 1 & 1 \\ 2 & 1 & 1 & 4 & 1 \\ 2 & 1 & 1 & 1 & 2 \end{bmatrix} ?$$

2 You might have already got some of the points below while staring at the matrix. The following facts are true. Tell for each why:

- The matrix has one single maximal eigenvalue.
- The matrix has real eigenvalues
- The matrix has zero as an eigenvalue
- The sum of the eigenvalues is known to be 16
- The determinant of A is zero
- $A - 3I_5$ has the vectors $[0, -1, 0, 1, 0]^T$ and $[0, -1, 1, 0, 0]^T$ in the kernel
- $A - 2I_5$ has the vector $[1, -1, 0, -1, 1]^T$ in the kernel

3 Now find all the eigenvalues of A .