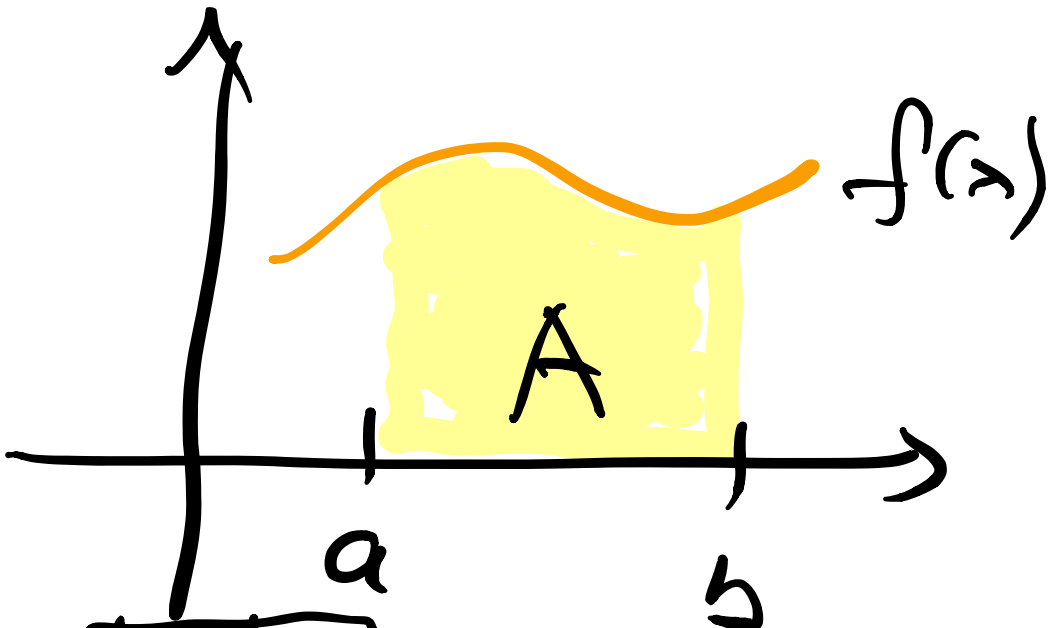
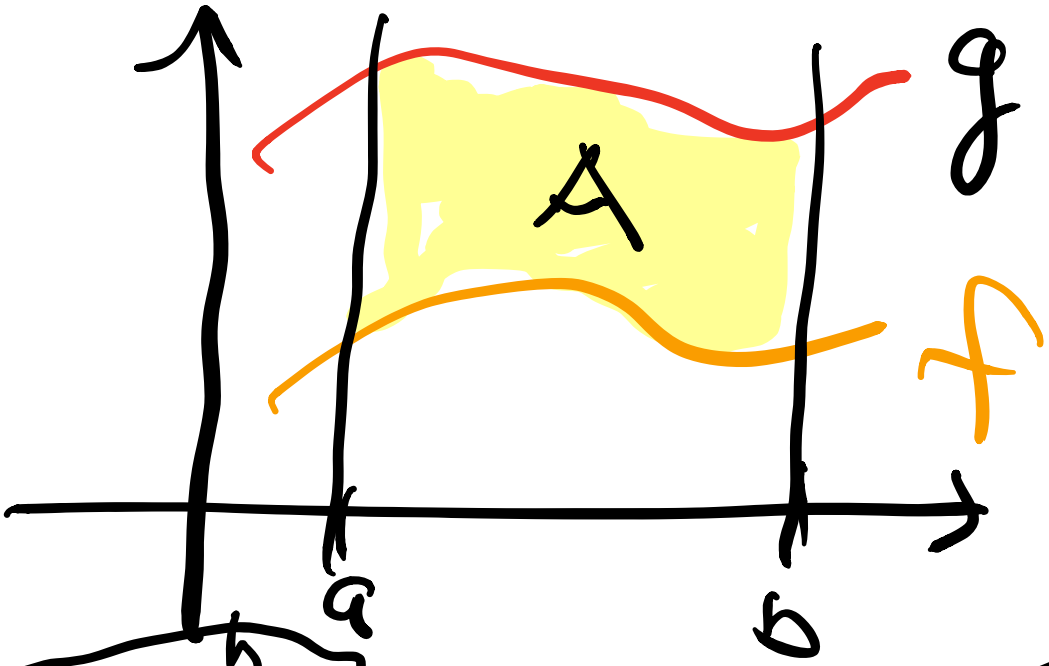
Unit 20

1) Area under the curve



$$\int_a^b f(x)\,dx = A$$

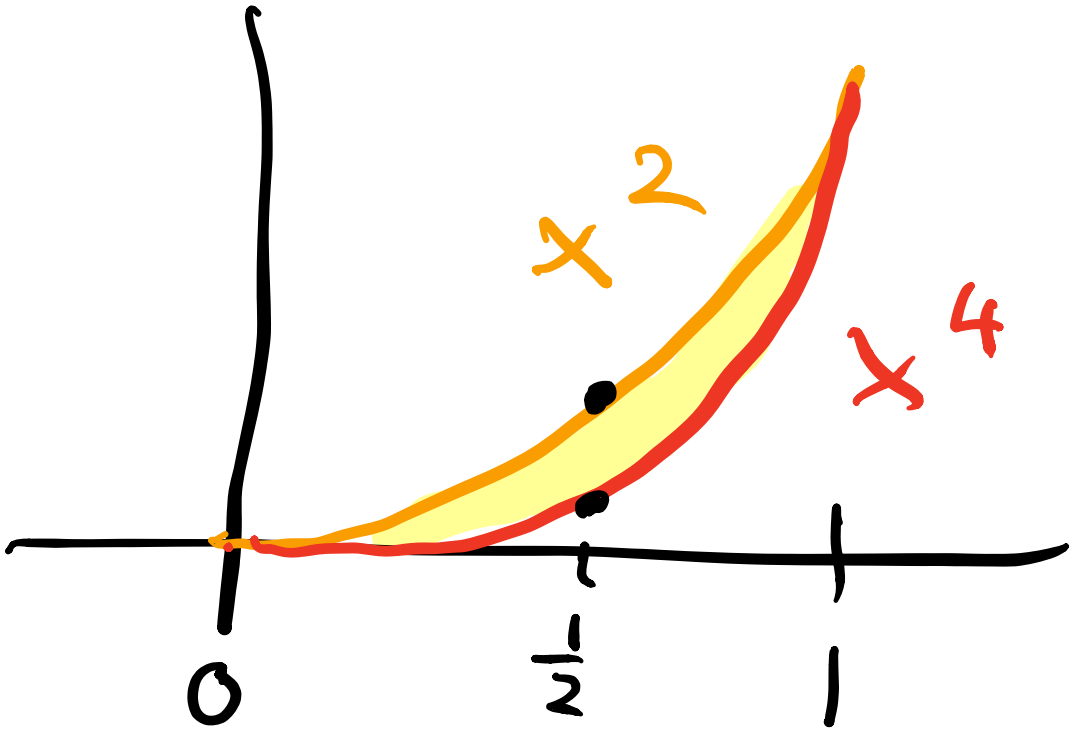
generalize:



$$A = \int_a^b g(x) - f(x)\ dx$$

Example: Area between x^2, x^4

on $[0,1]$



$$\int_0^1 x^2 - x^4 \, dx$$

$$= \frac{x^3}{3} - \frac{x^5}{5}\Big|_0^1$$

$$= \frac{1}{3} - \frac{1}{5} = \boxed{\frac{2}{15}}$$

Unit 20

Plan

1. Area under a curve
2. Area between 2 curves ✓
3. Examples
4. Circle
5. Shocker!
6. Ex

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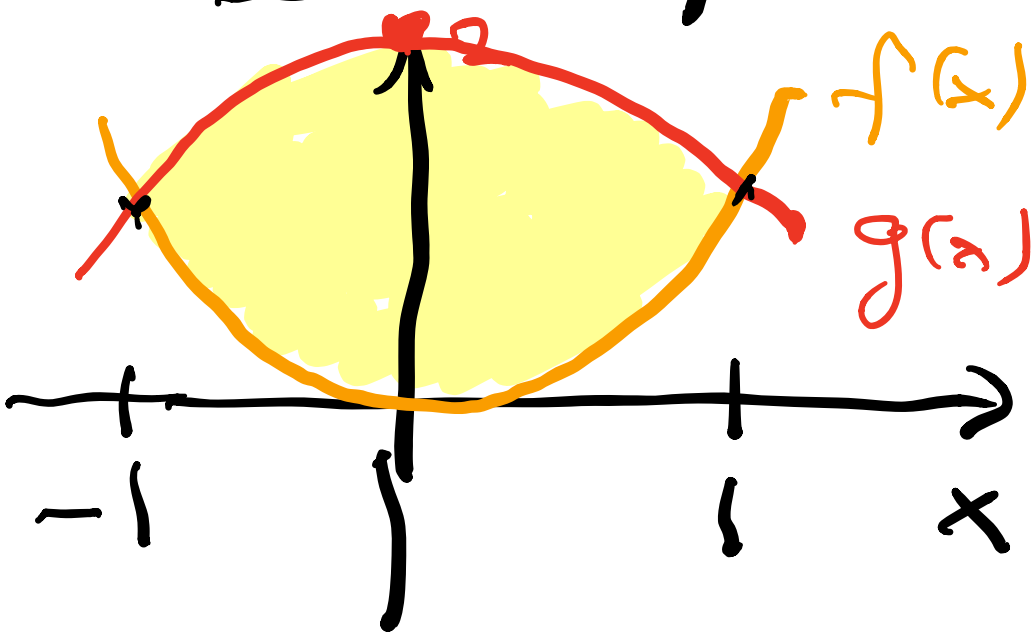
3 Examples

a) Find the area between $f(x) = x^6$ and $g(x) = 2 - x^6$

Make a picture:



$$x^6 = 2 - x^6$$

$$2x^6 = 2, \quad x^6 = 1$$

$$\int_{-1}^{1} (2 - x^6) - x^6 \, dx$$

$$\int_{-1}^{1} 2 - 2x^6 \, dx$$

$$= 2x - \frac{2x^7}{7}\Big|_{-1}^{1}$$

$$= \left(2 - \frac{2}{7}\right) \cdot 2$$

$$= \frac{12 \cdot 2}{7} = \boxed{\frac{24}{7}}$$

$$\sim 3.5$$

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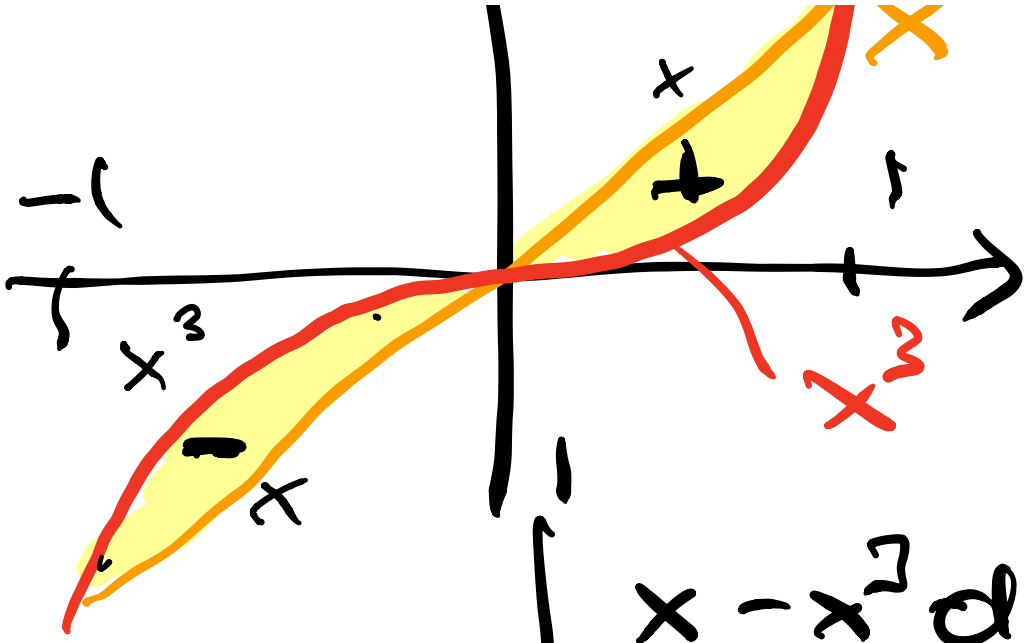
b) Propeller example

Find the area of the region bound by x^3, x



$$\int_{-1}^{1} x - x^3 \, dx$$

$$= 0$$

Wrong! What is off?

$$2\int_{0}^{1} x - x^3 \, dx$$

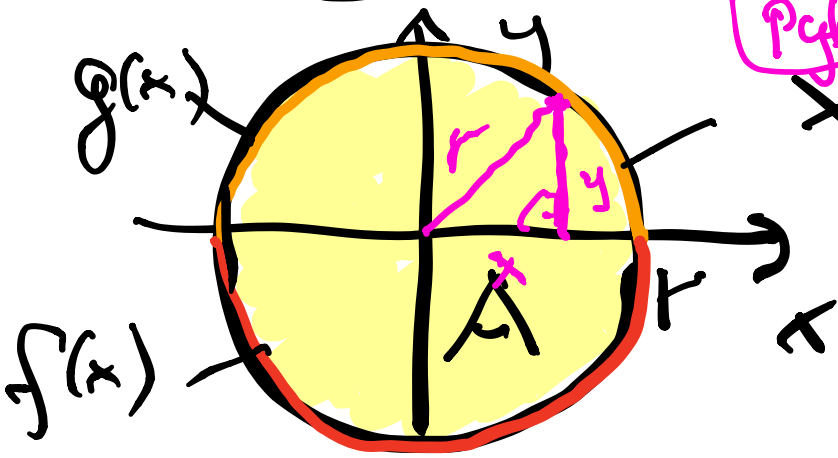
$$2\left(\frac{x^2}{2} - \frac{x^4}{4}\right)\Big|_0^1$$

$$\frac{1}{2}$$

(Homework)

make a picture.

(4) **Area of circle**



Pythagoras

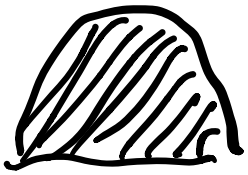
$x^2 + y^2 = r^2$

$y = \sqrt{-x^2 + r^2}$

$$f(x) = -\sqrt{r^2 - x^2}$$

$$g(x) = \sqrt{r^2 - x^2}$$

$$2\int_{-r}^{r} \sqrt{r^2 - x^2}\, dx$$



difficult integral

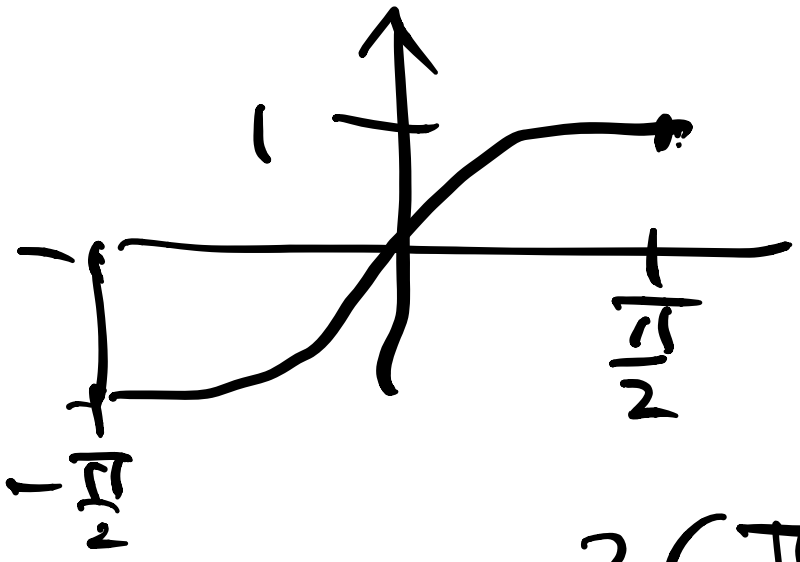
$f(x) = 2\sqrt{r^2 - x^2}$ has the antiderivative:

$$\boxed{x\sqrt{r^2 - x^2} + r^2 \arcsin\left(\frac{x}{r}\right)}$$

evaluate at $x = r$

$x = -r$

$$r^2\left(\arcsin(1) - \arcsin(-1)\right)$$



$$= r^2\left(\frac{\pi}{2} - \left(-\frac{\pi}{2}\right)\right)$$

$= \boxed{r^2\pi}$

tough problem.
We need more
techniques.

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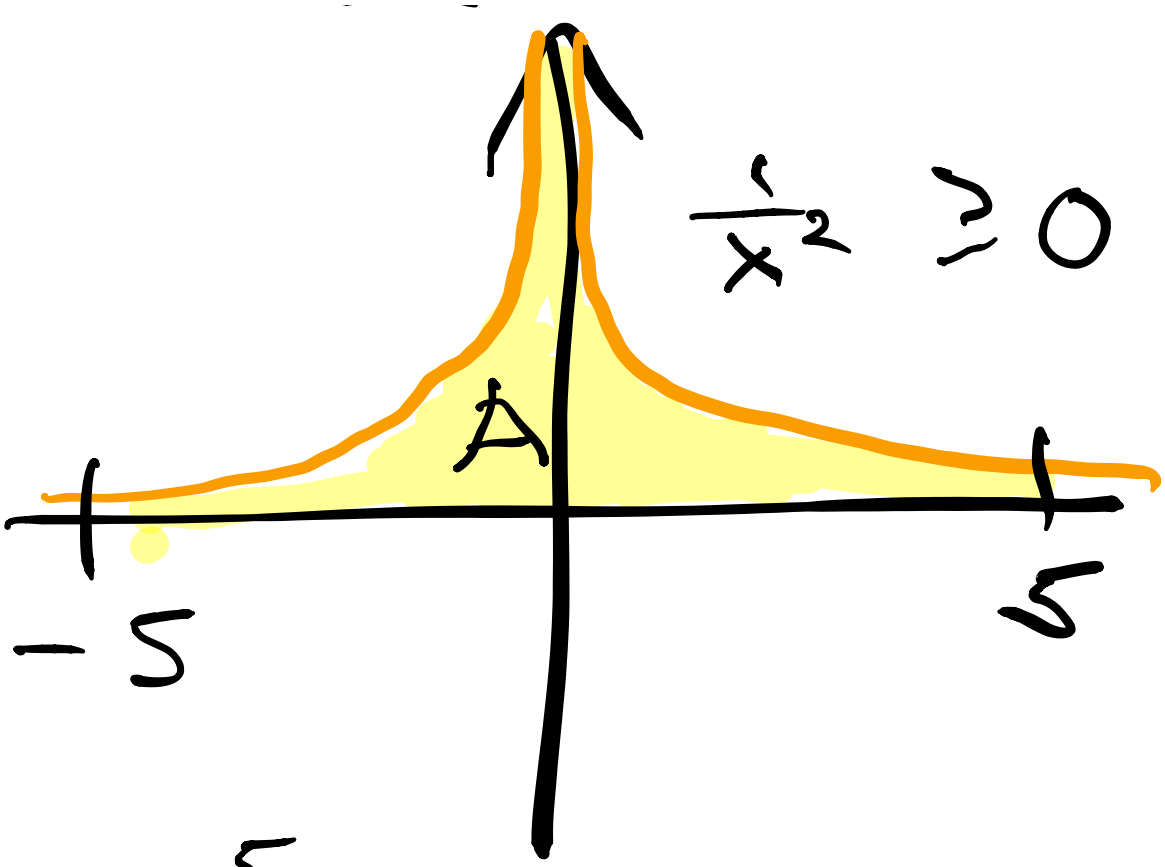
(5) **Shocking example**

Find the area
under the curve
$f(x) = \frac{1}{x^2}$
where $-5 \leq x \leq 5$



$$\int_{-5}^{5} \frac{1}{x^2}\, dx$$

$$= -\frac{1}{x}\Big|_{-5}^{5}$$

$$-\frac{1}{5} - \frac{1}{5} = \boxed{-\frac{2}{5}}$$

Shock:

Area should be positive

Learn: We have to watch out for discontinuities

$\frac{1}{\quad}$ is not

x^2 continuous

Improper integral

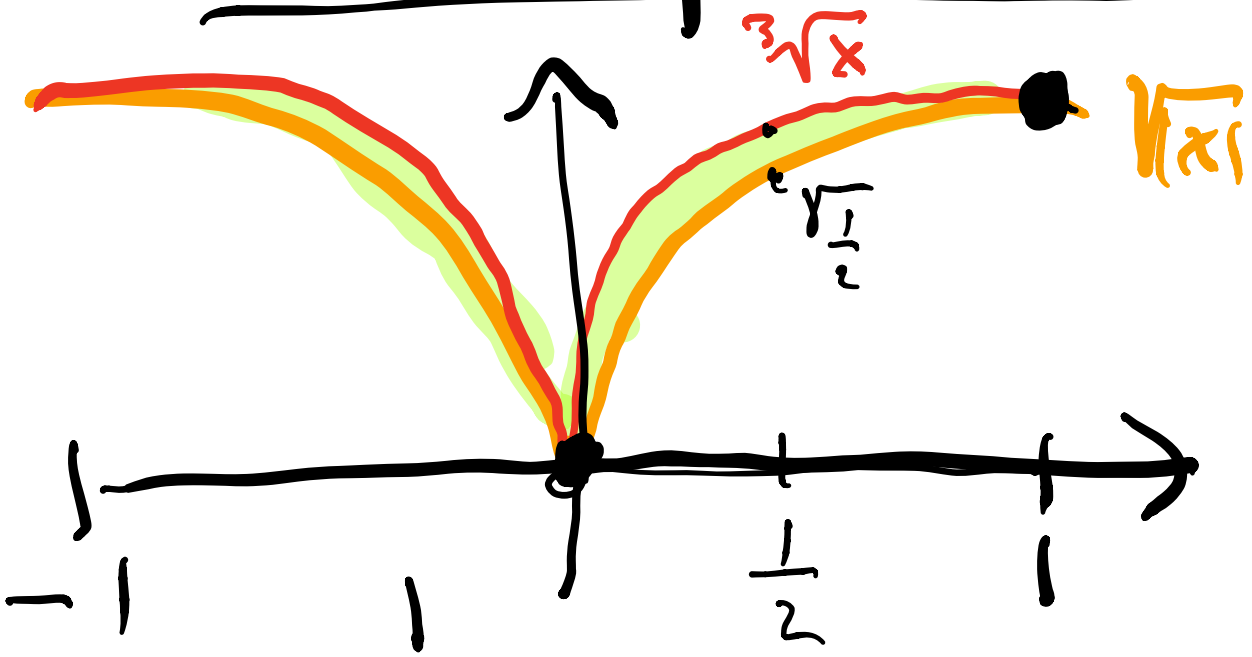
⑥ Example

Grass

Find the

area between $|x|^{1/3}$ and $|x|^{1/2}$

make a picture!



$$2\int_0^1 x^{1/3} - x^{1/2}\,dx$$

$$2\left(\frac{3}{4}x^{4/3} - \frac{2}{3}x^{3/2}\right)\Big|_0^1$$

$$2\left(\frac{3}{4}-\frac{2}{3}\right)=\frac{2}{12}$$

$$=\boxed{\frac{1}{6}}$$