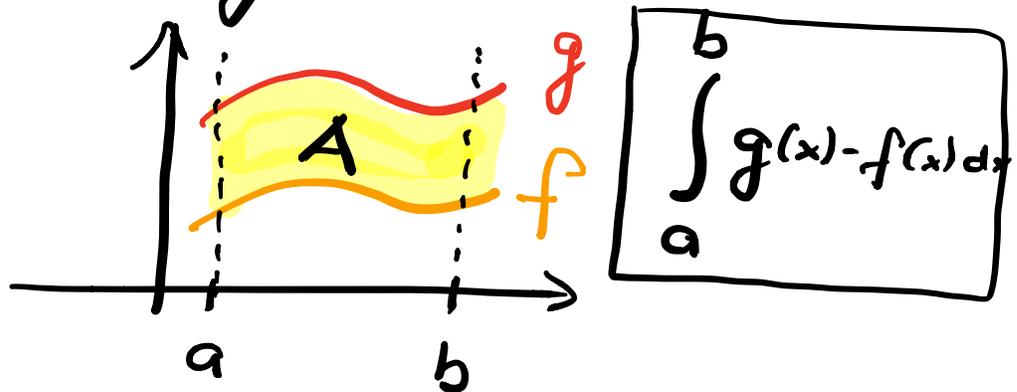


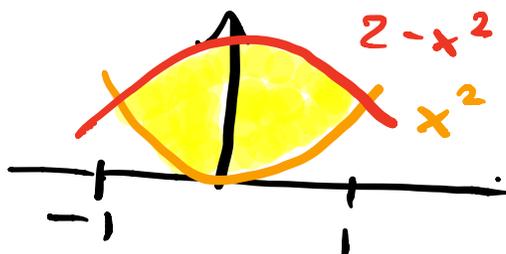
Unit 20 Area

1. Integral and area



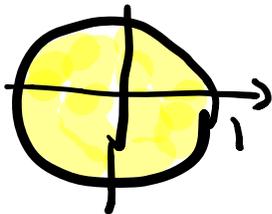
2. Examples

a) Area bound by $x^2, 2-x^2$



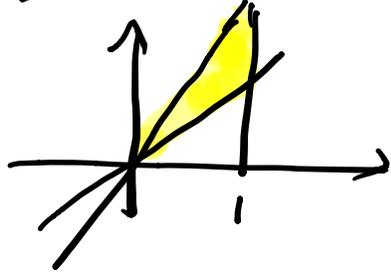
$$\int_{-1}^1 2 - 2x^2 dx$$
$$= 2x - \frac{2x^3}{3} \Big|_{-1}^1$$
$$= 4 - \frac{4}{3} = \frac{8}{3}$$

b) Area of disc



$$\int_{-1}^1 \sqrt{1-x^2} dx =$$
$$\frac{x\sqrt{1-x^2}}{2} + \arcsin x \Big|_{-1}^1$$
$$= \pi$$

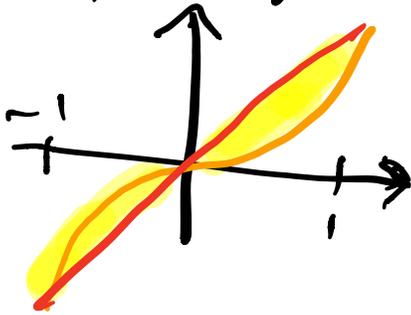
c) Area bound by $y=x, y=2x$



$$x=1$$

$$\int_0^1 x dx = \boxed{\frac{1}{2}}$$

d) Propeller (remember?)

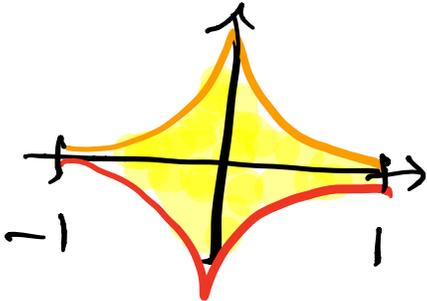


$$2 \int_0^1 x - x^3 dx$$

$$= 2 \left(\frac{x^2}{2} - \frac{x^4}{4} \right) \Big|_0^1$$

$$= \boxed{\frac{1}{2}}$$

e) Area bound by $1-(x)^{1/4}$
 $-1+(x)^{1/4}$

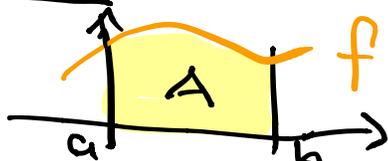


$$2 \cdot 2 \int_0^1 1 - x^{1/4} dx$$

$$= 4 \left(x - x^{5/4} \cdot \frac{4}{5} \right) \Big|_0^1$$

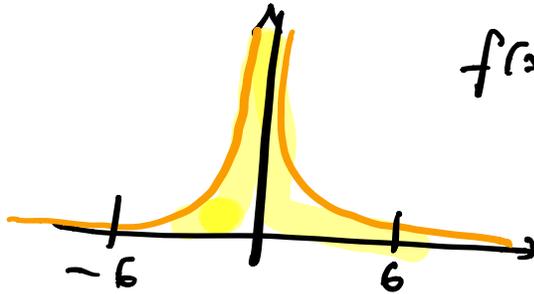
$$= \boxed{\frac{4}{5}}$$

Remember: It all started with



$$\int_a^b f(x) dx = A$$

f) A paradox



$$f(x) = \frac{1}{x^2}$$

$$\int_{-6}^6 \frac{1}{x^2} dx$$

$$= -\frac{1}{x} \Big|_{-6}^6$$

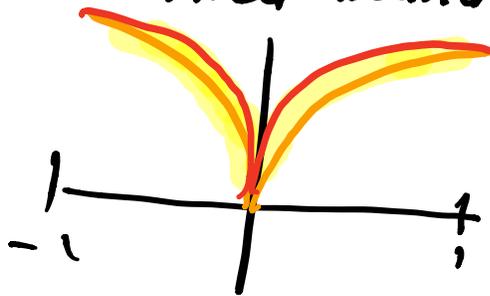
$$= -\frac{1}{6} - \frac{1}{-6} = -\frac{1}{3}$$

What is wrong?

g) Grass problem

Area bound by

$$|x|^{1/3}, |x|^{1/2}$$



$$x = \frac{1}{2}, x^{1/3} = \frac{1}{\sqrt[3]{2}} > x^{1/2}$$

$$2 \int_0^1 x^{1/3} - x^{1/2} dx$$

$$= 2 \left(\frac{3}{4} x^{4/3} - \frac{2}{3} x^{3/2} \Big|_0^1 \right)$$

$$= 2 \left(\frac{3}{4} - \frac{2}{3} \right) = \frac{2}{12} = \frac{1}{6}$$