Unit 21 | Volume
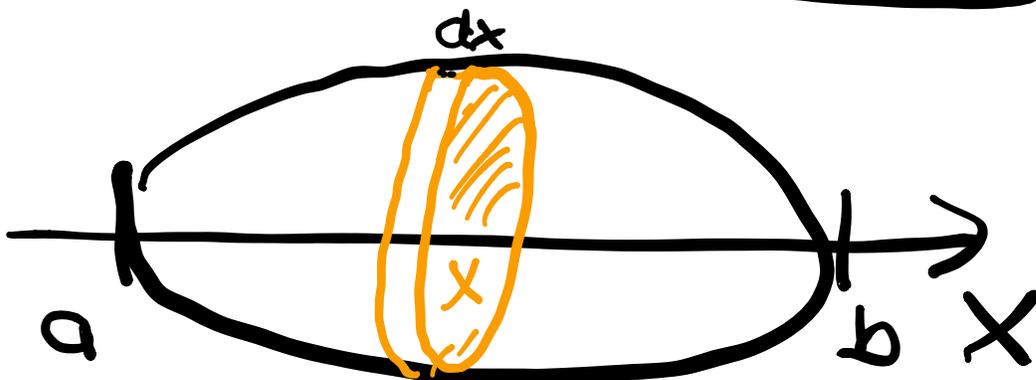
Plan:

1) Volume formula
2) Cone + pyramid
3) Sphere (Archimedes)
4) Video of lemon rasp.
5) Pineapple execution
6) Breakout room.

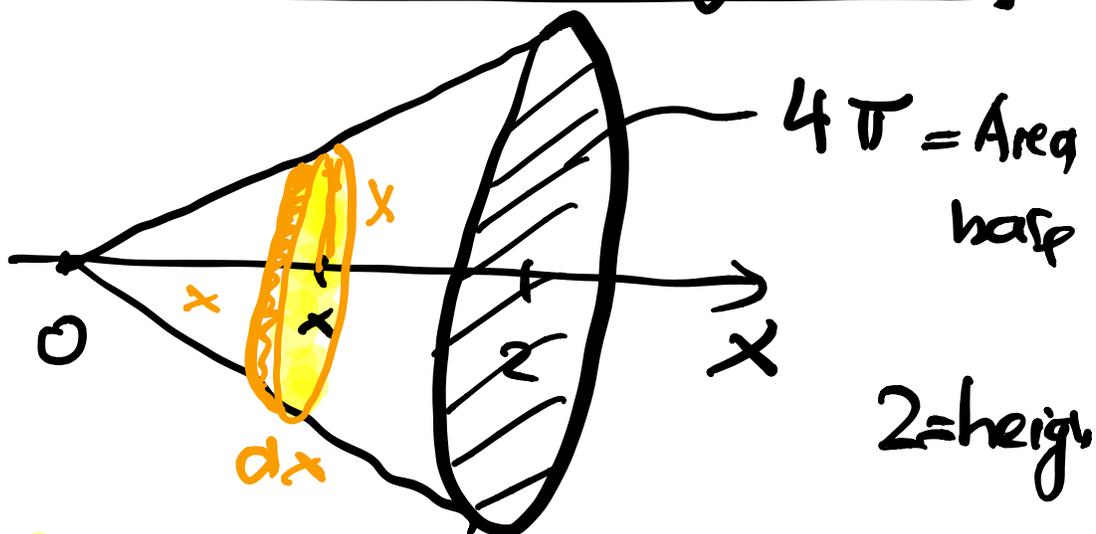① | Volume computation |

dx

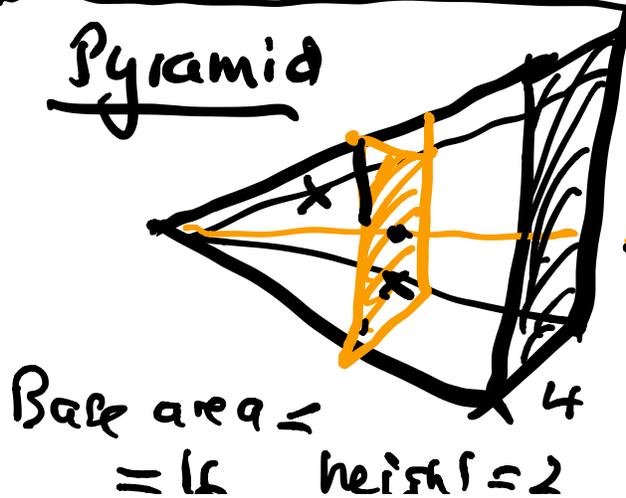$A(x)$ Area of slice

$$\int_a^b A(x)\, dx = \underline{Volume}$$

② **Cone and pyramid**

$4\pi = $ Area base

$2 = $ heigh

$A(x) = x^2 \pi$

$$\int_0^2 x^2 \pi \, dx = \frac{x^3}{3} \pi \Big|_0^2$$

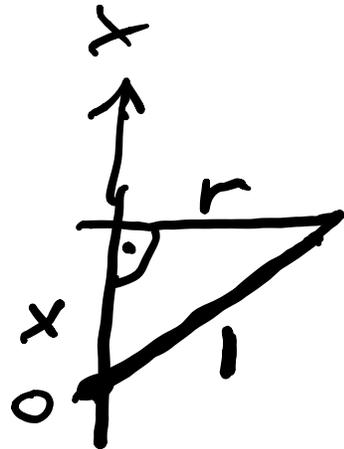$$= \boxed{\frac{8\pi}{3}} = \text{Base A} \cdot \text{heigh}/_3$$
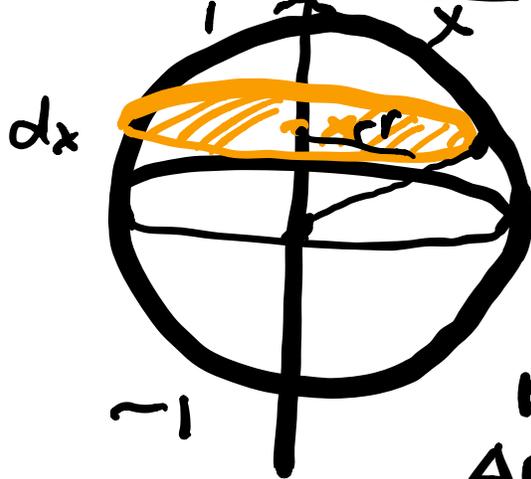
Pyramid

Base area =
= 16   heighl = 2

$A(x)$

$2 = 4x^2$ $\boxed{32/_3}$

$$\int_0^2 4x^2 \, dx = \frac{8}{3} \cdot 4$$

③ ┌─────────┐
   │ Sphere │
   └─────────┘

dx

1

r x

-1

$r = \sqrt{1-x^2}$

$A(x) = \pi(1-x^2)$

$$\int_{-1}^{1} \pi(1-x^2)\, dx = \pi \int_{-1}^{1} 1-x^2\, dx$$
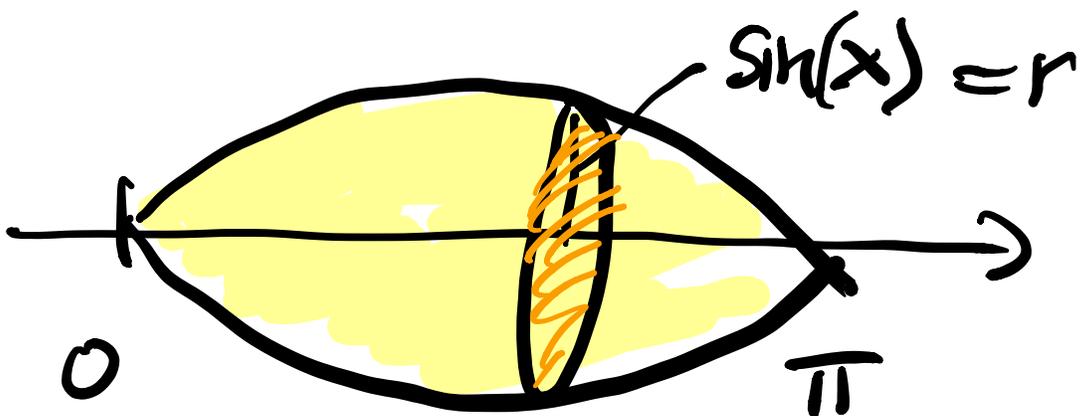
$$= \pi \left(x - \frac{x^3}{3} \right) \Big|_{-1}^{1}$$

$$= \pi \left(\frac{2}{3} + \frac{2}{3} \right) = \boxed{\frac{4\pi}{3}}$$

πx^2

$A(x)$

$A(x)$

$\pi - \pi x^2$

$$\frac{\text{vol (cylinder)} - \text{vol (cone)}}{\text{remember Archimedes}}$$
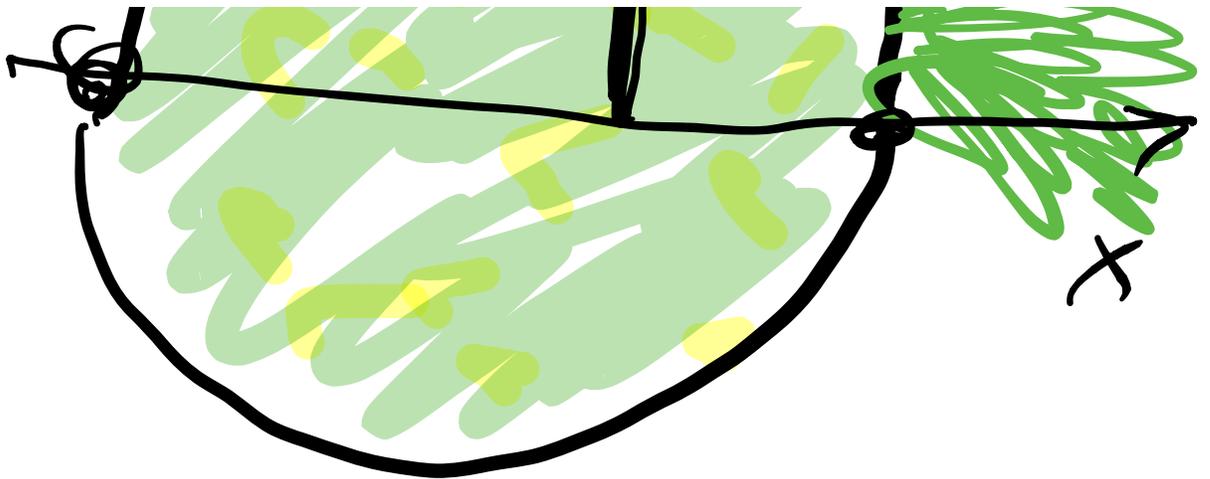
(f) Lemon

$\sin(x) = r$

0 π

$$A(x) = \pi \sin^2 x$$

$$\int_0^\pi \pi \sin^2 x \, dx$$

Double angle formula

$$\sin^2 x = \frac{1 - \cos 2x}{2}$$

$$= \frac{1}{2} - \frac{\cos 2x}{2}$$

$$\pi \int_0^\pi \frac{1}{2} - \frac{\cos 2x}{2}\, dx$$

$$\pi \left(\frac{x}{2} - \frac{\sin 2x}{4} \right) \Big|_0^\pi$$

$$= \boxed{\frac{\pi^2}{2}} = \underline{\text{Volume}}$$

5) | Pine apple |

$$r = \sqrt{81 - x^4}$$

$$81 - x^4 = 0$$

$$x = \pm 3$$

243 π $\frac{8}{5}$

$$\pi \int_{-3}^{3} (81 - x^4) dx$$

$$= \left(81x - \frac{x^5}{5}\right)\Big|_{-3}^{3} \pi$$