Unit 21 Volume

1. Set-up
2. Cone
3. Sphere
4. Lemon
5. Catenoid

① Set-up

$A(x) =$ Area

$$\text{Volume} = \int_a^b A(x)\,dx$$

Think about $dV = A(x)\,dx$ as a slice.

Adding up all the slices gives the volume.

[We can use calculus, to compute volumes.!

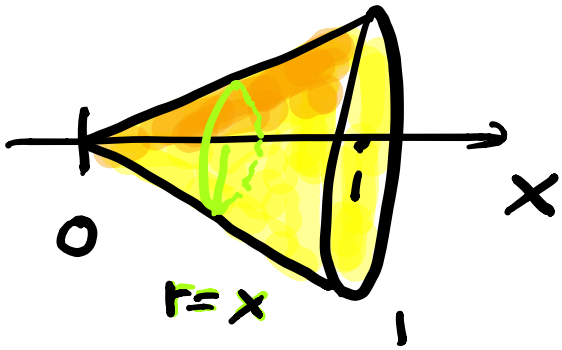
2. **The cone**

cone :

radius at x is equal to x.

Area at x is equal to $x^2\pi$

AGJ



$$\text{Volume} = \int_0^1 x^2\pi\,dx = \frac{x^3}{3}\pi\Big|_0^1 = \frac{\pi}{3}$$

More general :

$$\frac{\text{Base area} \times \text{height}}{3}$$

3. **The sphere**

Volume of sphere : $\frac{4\pi}{3}$!

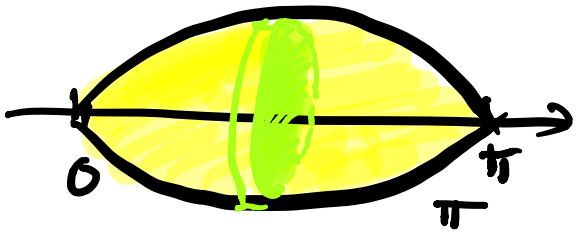
$r = \sqrt{1-x^2}$

$$\text{Area}(x) = r^2\pi = (1-x^2)\pi$$

$$\text{Volume} = \int_0^1 (1-x^2)\pi\,dx = \left(x - \frac{x^3}{3}\right)\Big|_0^1 \pi$$

half sphere $= \frac{2}{3}\pi$

4. **Lemon**



$r(x) = \sin x$

$0 \le x \le \pi$

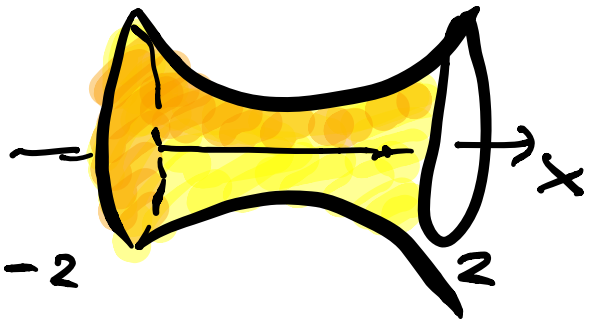
$A(x) = \sin^2(x)\pi$

$$\text{Volume} = \int_0^{\pi} \pi \sin^2 x \, dx$$

$$= \int_0^{\pi} \pi \left(\frac{1-\cos(2x)}{2}\right) dx$$

$$= \frac{\pi}{2}\left(x - \frac{\sin(2x)}{2}\right)\Big|_0^{\pi} = \frac{\pi^2}{2}$$

5. **Catenoid**



$r(x) = \cosh x$

$= \frac{e^x + e^{-x}}{2}$

→ Homework