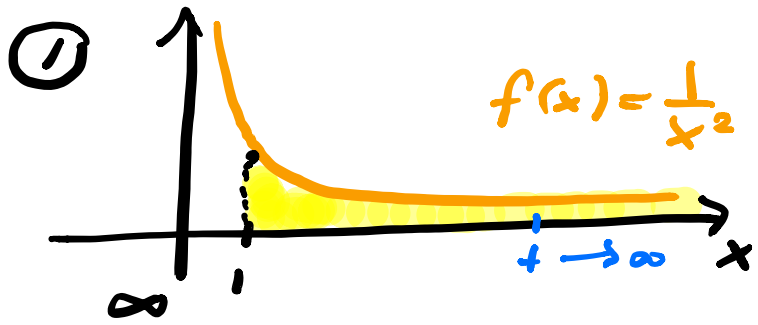
Plan:

1. Finite or not
2. Shock reorew
3. Examples
4. Making Sense of Zeno
5. Gabriels trumpet

Unit 22 Improper integrals



Does $\int_1^\infty \frac{1}{x^2}\,dx$ make sense?

To find out, compute

$$\int_1^t \frac{1}{x^2}\,dx = \frac{-1}{x}\Big|_1^t = 1-\frac{1}{t}$$

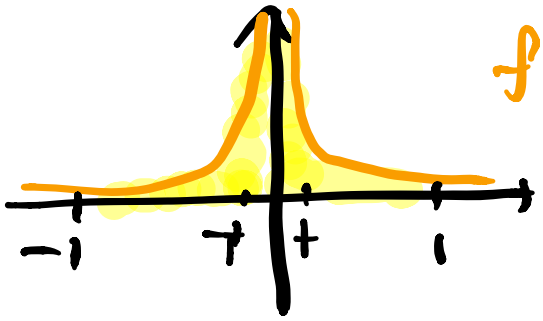
and look where the limit $t\to\infty$ exists. Yes it does.

What about

$$\int_1^\infty \frac{1}{x}\,dx\ ?$$

Again, look at $\int_1^t \frac{1}{x}\,dx$ $= \log t$ and look at the limit $t\to\infty$ this does not work.

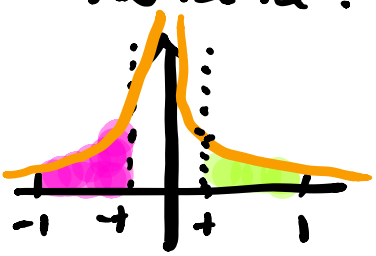
2. Shock example



$$\int_{-1}^{1} \frac{1}{x^2}\,dx$$

blind: $= -\frac{1}{x}\Big|_{-1}^{1} = -2$

Nonsense!



$$\int_{-1}^{-t} \frac{1}{x^2}\,dx = -\frac{1}{x}\Big|_{-1}^{-t} = -1+\frac{1}{t}$$

$$\int_{+t}^{1} \frac{1}{x^2}\,dx = -\frac{1}{x}\Big|_{+t}^{1} = -1+\frac{1}{t}$$

The integral does not exist!

what about $\int_{-1}^{1} \frac{1}{\sqrt{x}}\,dx$?

$$\int_{+t}^{1} \frac{1}{\sqrt{x}}\,dx$$

$$= 2\sqrt{x}\Big|_{+t}^{1} = 2-2\sqrt{t} \xrightarrow[+t\to 0]{} 2$$

exists

So, we have

③ Examples

a) $\int_1^\infty \frac{1}{x^3}\,dx$

b) $\int_0^\infty e^{-x}\,dx$

c) $\int_0^1 \frac{1}{1+x^2}\,dx$

d) $\int_0^1 x^{3/4}\,dx$

e) $\int_1^\infty x^{3/4}\,dx$

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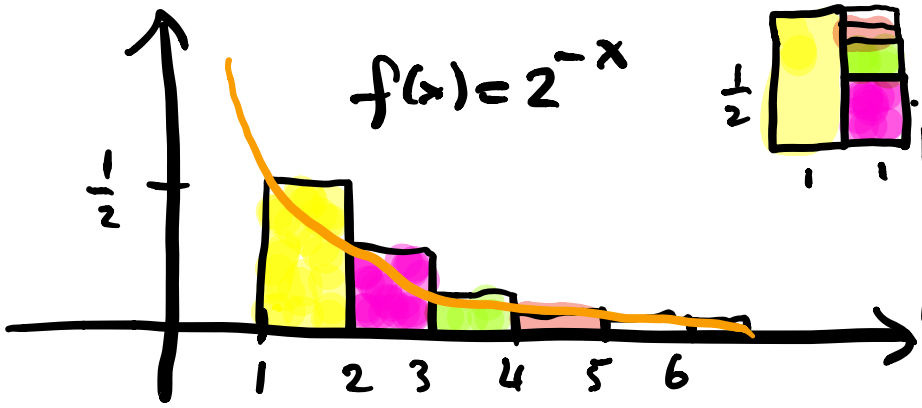
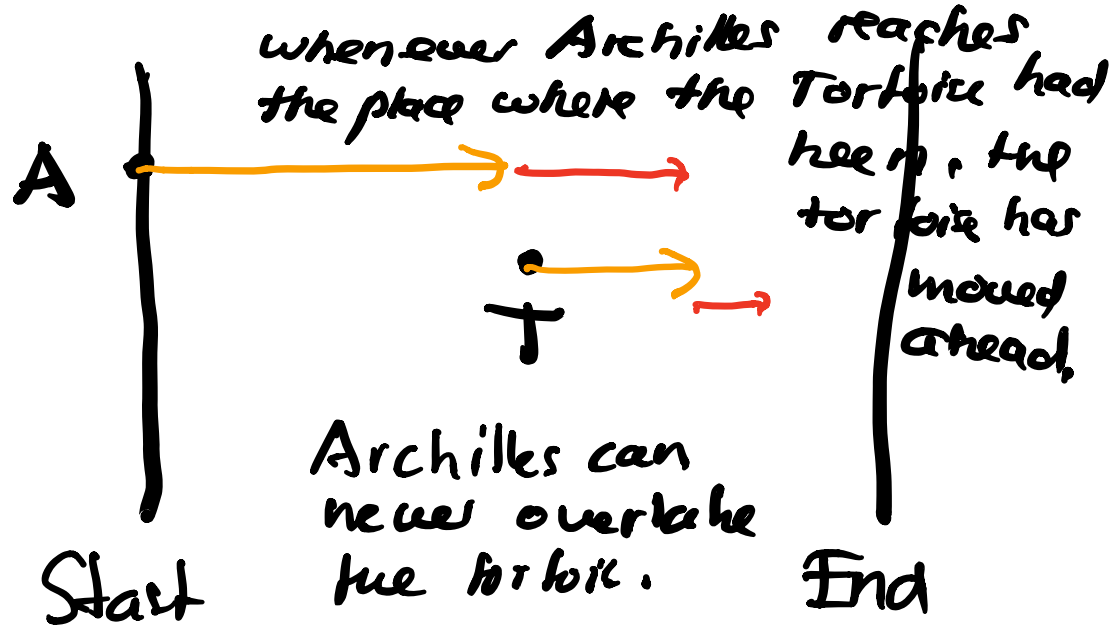
4) How to make sense ?



$$-\frac{2^{-x}}{\log 2}\Big|_1^\infty = \frac{1}{2\log 2} \approx 0.72$$

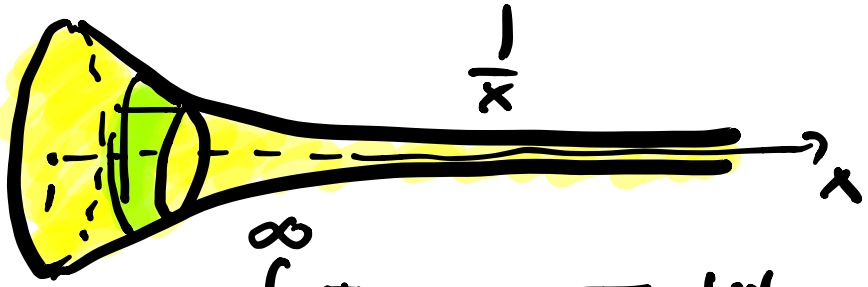
$$\int_1^\infty 2^{-x}\,dx < 1$$

Zeno : Archilles and tortoise



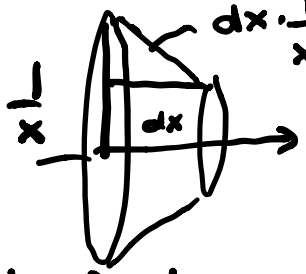
5.

Trumpet



$$\int_1^\infty \frac{\pi}{x^2}\,dx = \pi$$ Volume finite

Surface area $\geq \int_1^\infty \frac{1}{x}\,dx$



$$= \lim_{t\to\infty} \int_1^t \frac{1}{x}\,dx$$

$$= \lim_{t\to\infty} \log t \longrightarrow \infty$$

Infinite surface area

We can fill the trumpet! But not paint it.