Unit 25
Integration by parts

1. Method
2. Examples
3. Repeated
4. Merry go round
5. Tic - Tac - Toe
6. Stand and deliver clip

$$du = u' dx$$

$$\frac{du}{dx} = u'$$

① **Method**   Integrate:

$$(uv)' = v u' + u v'$$

$$uv = \int v\, du + \int u\, dv$$

$$\int u\, dv = uv - \int v\, du$$

$$\int x \cos x\, dx = x \sin x - \int \sin x \cdot 1\, dx$$

$$= x \sin x + \cos x + C$$

② | Examples |

a) $\int 2x \, e^{5x} \, dx$

$$= 2x \, \frac{e^{5x}}{5} - \int 2 \cdot \frac{e^{5x}}{5} \, dx$$

$$= \boxed{2x \, \frac{e^{5x}}{5} - \frac{2}{25} e^{5x} + C}$$

b) $\int \log x \cdot 1 \, dx$

$$= \log x \cdot x - \int \frac{x}{x} \, dx$$

$$= \boxed{\log x \cdot x - x + C}$$

c) $\int \log x \cdot x \, dx$

$$\log x \, \frac{x^2}{2} - \int \frac{x^2}{2} \frac{1}{x} \, dx \qquad = \frac{x}{2}$$

Do on your own

LIATE

$$\boxed{\log x \, \frac{x^2}{2} - \frac{x^2}{4} + C}$$

③ **Repeated application**

$$\int x^2 \cos x \, dx$$

$$x^2 \sin x - \int \sin x \, 2x \, dx$$

$$x^2 \sin x - \left(-\cos x \cdot 2x \right.$$
$$\left. - \int -\cos x \cdot 2 \, dx \right)$$

$$= x^2 \sin x + 2x \cos x - \int \cos x \cdot 2 \, dx$$

$$= \boxed{x^2 \sin x + 2x \cos x - 2 \sin x + C}$$

A lot of work. → Tic Tactor
Better method.

→ At the end of this class.

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④ **Merry-go-round**

$$I = \int \sin x \, e^x \, dx$$

$$= \sin x \, e^x - \int e^x \cos x \, dx$$

$$= \sin x \, e^x - e^x \cos x$$

$$+ \int e^x (-\sin x) \, dx$$

$$= \sin x \, e^x - e^x \cos x - I$$

we moved in circles!

Solve for I !

$$2I = \sin x \, e^x - e^x \cos x$$

$$I = \frac{\sin x \, e^x - e^x \cos x}{2} + c$$

$$I = \int \tan(x) \frac{1}{\cos^2 x} \, dx$$

$$\left(\begin{array}{l} \text{This was solved by} \\ \text{substitution} \quad u = \tan x \end{array} \right)$$

$$= \frac{\tan(x) \, \tan(x)}{} - \int \frac{\tan x}{\cos^2 x} \, dx$$

$$= \tan^2(x) - I$$

$$2I = \tan^2(x), \quad \boxed{I = \frac{\tan^2 x}{2} + c}$$

We got that

by substitution.

(5) **Tic-tac-toe**

a) $\int x^2 \cos x \, dx$

$$x^2 \sin x$$
$$+ 2x \cos x$$
$$- 2 \sin x$$
$$+ C$$

x^2	$\cos x$	
$2x$	$\sin x$	$+$
2	$-\cos x$	$-$
0	$-\sin x$	$+$

differen   Integr.   Signs

Answer:

$$x^2 \sin x - \int 2x \sin x \, dx$$

b) $\int x^5 e^{3x} \, dx$

$$x^5 e^{3x}/3$$
$$- 5x^4 e^{3x}/9$$
$$+ 20x^3 e^{3x}/27$$
$$- 60x^2 e^{3x}/81$$
$$+ 120x\, e^{3x}/243$$
$$- 120\, e^{3x}/729$$
$$+ c$$

x^5	e^{3x}	Sign
$5x^4$	$e^{3x}/3$	$+$
$20x^3$	$e^{3x}/9$	$-$
$60x^2$	$e^{3x}/27$	$+$
$120x$	$e^{3x}/81$	$-$
120	$e^{3x}/243$	$+$
0	$e^{3x}/729$	$-$