Unit 25 Parts

① How does it work?

$$\int x \cos(x)\, dx$$

$$= x \sin x - \int \sin x\, dx$$

$$= \boxed{x \sin x + \cos x + C}$$

Justification:

$$\int \underset{u}{x}\ \underset{dv}{\cos x\, dx} = uv - \int v\, du$$

$uv = x \sin x$, $v\, du = \int \sin x\, dx$

② Why does it work

Product rule

$$(uv)' = u'v + uv'$$

$$uv = \int u'v + \int uv'$$

$$\int u\, dv = uv - \int v\, du$$

③ Examples

a) $\int x e^x\, dx = x e^x - \int e^x\, dx$

$$= x e^x - e^x + C$$

b) $\int \log x \cdot 1 \, dx = \log x \cdot x - \int \frac{x}{x} \, dx$

$= \log x \cdot x - x + C$

---

c) $I = \int e^x \sin x \, dx = e^x \sin x - \int e^x \cos x \, dx$

$\int e^x \cos x \, dx = e^x \cos x + \int e^x \sin x \, dx$

combine:

$I = e^x \sin x - e^x \cos x - I$

$2I = e^x(\sin x - \cos x)$

$I = \frac{e^x(\sin x - \cos x)}{2} + C$

---

④ Tic-Tac-Toe

$\int x^2 \sin x \, dx$

x^2	$\sin x$	
$2x$	$-\cos x$	$+$
2	$-\sin x$	$-$
0	$\cos x$	$+$

$\boxed{-x^2 \cos x + 2x \sin x + 2\cos x + C}$

1988

→ Movie Stand and deliver

$$\int x^4 e^x \, dx$$

x^4	e^x	
$4x^3$	e^x	+
$12x^2$	e^x	−
$24x$	e^x	+
24	e^x	−
0	e^x	+

$$\boxed{\left(x^4 - 4x^3 + 12x^2 - 24x + 24\right) e^x}$$

---

⑤ Mary go round

$$\int \sin x \cos x \, dx$$

$$\sin^2 x - \int \cos x \sin x \, dx$$

$$\Rightarrow \quad 2 \sin x \cos x = \sin^2 x$$

$$\sin x \cos x = \frac{\sin^2 x}{2}$$

$$I = \int \sqrt{1+x^2} \, dx = x\sqrt{1+x^2} - \int \frac{x^2}{\sqrt{1+x^2}} \, dx$$

$$= x\sqrt{1+x^2} + \int \frac{1}{\sqrt{1+x^2}} \, dx - \int \sqrt{1+x^2}$$

$$2I = x\sqrt{1+x^2} - \operatorname{arcsinh}(x)$$

$$I = \boxed{\frac{x\sqrt{1+x^2} + \operatorname{arcsinh}(x)}{2} + c}$$