

Unit 25 Parts

① How does it work?

$$\begin{aligned} & \int x \cos(x) dx \\ & \quad \downarrow \quad \uparrow \\ & = x \sin x - \int \sin x dx \\ & = \boxed{x \sin x + \cos x + C} \end{aligned}$$

Justification:

$$\int u dv = uv - \int v du$$

$u v = x \sin x, v du = \int \sin x dx$

② Why does it work

Product rule $(uv)' = u'v + uv'$

$$uv = \int u'v + \int uv'$$
$$\int u dv = uv - \int v du$$

③ Examples

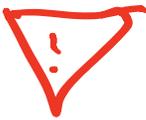
a) $\int x e^x dx = x e^x - \int e^x dx$

$$= \boxed{x e^x - e^x + C}$$

$$b) \int \log x \cdot 1 \, dx = \log x \cdot x - \int \frac{x}{x} \, dx = \log x \cdot x - x + C$$

$$c) I = \int e^x \sin x \, dx = e^x \sin x - \int e^x \cos x \, dx$$

$$\int e^x \cos x \, dx = e^x \cos x + \int e^x \sin x \, dx$$



combine:

$$I = e^x \sin x - e^x \cos x - I$$

$$2I = e^x (\sin x - \cos x)$$

$$I = \frac{e^x (\sin x - \cos x)}{2} + C$$

④ Tic-Tac-Toe

$$\int x^2 \sin x \, dx$$

x^2	$\sin x$	
$2x$	$-\cos x$	$+$
2	$-\sin x$	$-$
0	$\cos x$	$+$

$$\begin{aligned} & -x^2 \cos x \\ & + 2x \sin x \\ & + 2 \cos x \\ & + C \end{aligned}$$

1988

→ Movie Stand and deliver

$$\int x^4 e^x dx$$

x^4	e^x	
$4x^3$	e^x	+
$12x^2$	e^x	-
$24x$	e^x	+
24	e^x	-
0	e^x	+

$$\begin{aligned} & (x^4 - 4x^3 \\ & + 12x^2 - 24x \\ & + 24) e^x \end{aligned}$$

⑤ May go round

$$\int \sin x \cos x dx$$

\downarrow \uparrow
 $\sin^2 x$ $\cos x$

$$\sin^2 x - \int \cos x \sin x dx$$

$$\Rightarrow 2 \sin x \cos x = \sin^2 x$$

$$\sin x \cos x = \frac{\sin^2 x}{2}$$

$$I = \int \sqrt{1+x^2} dx = x \sqrt{1+x^2} - \int \frac{x^2}{\sqrt{1+x^2}} dx$$

$$= x \sqrt{1+x^2} + \int \frac{1}{\sqrt{1+x^2}} dx$$

$$- \int \frac{1}{\sqrt{1+x^2}} dx$$

$$2I = x \sqrt{1+x^2}$$

$$- \operatorname{arcsinh}(x)$$

$$I = \frac{x \sqrt{1+x^2} + \operatorname{arcsinh}(x)}{2} + C$$

