Unit 26 Review

① fc

$f' = 0$

Catastrophes

Catastrophe: c value where # of minima changes

Stable equilibria = minima

$f'' > 0$ for minima ← $f'' = 0$

5 $\int f(x)\,dx$ = probab

PDF

Piecewise cond. ≥ 0

$\int_{-\infty}^{\infty} f(x)\,dx = 1$

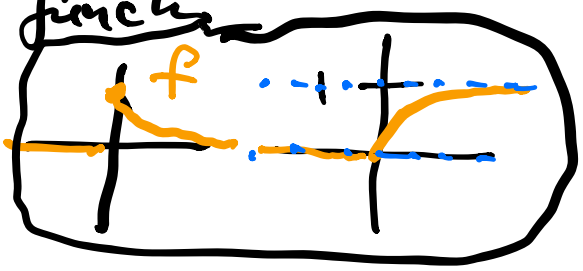
Stats

CDF

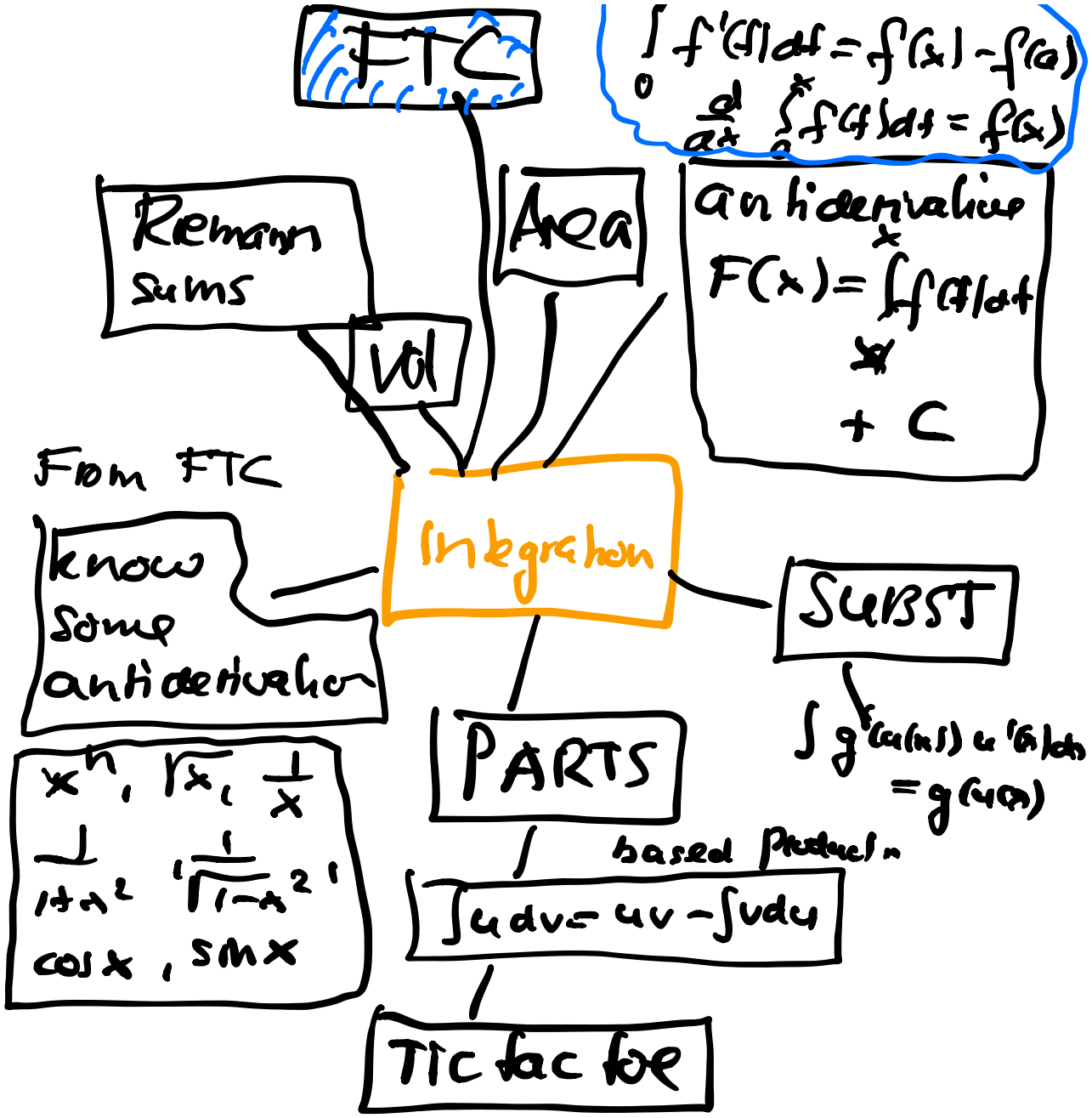
$F = \int_{-\infty}^{t} f(t)\,dt$

normal exponential uniform cauchy

monotone func



Integration



$$\int \underbrace{g'(u(x))\,u'(x)\,dx}_{du} \overset{\text{chain-rule}}{=} g(u) + C$$

$$\int g'(u)\,du = g(u) + C$$

(2) Problems

a) $f_c(x) = x^3 + cx$

Which parameters c are catastrophes?

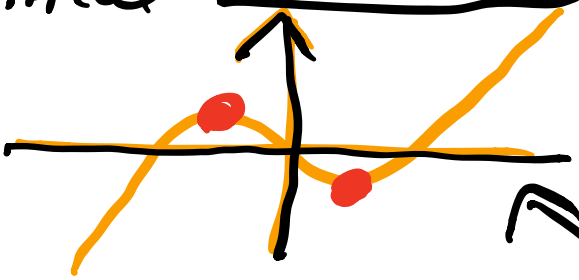
Task! What are the minima if c is given?

$$f_c'(x) = 3x^2 + c = 0$$

$$x_c = \pm\sqrt{\frac{-c}{3}}$$

$f''(x) = 6x$

two critical points



$$f_c''(x_c) = \pm 6\sqrt{\frac{-c}{3}} \qquad c < 0$$

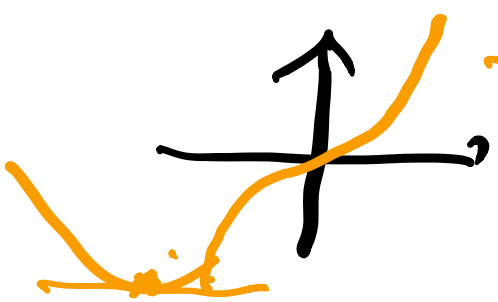
For which c do f_c have critical points?

c must be negative!

$x_c = \pm\sqrt{\frac{-c}{3}}$ has a positive

$f''(x_c)$ and therefore is a min

For $c > 0$, there is no critical point.



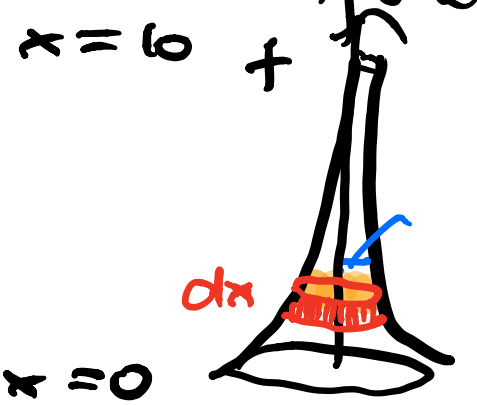
There is a catastrophe at $\boxed{c = 0}$

For $c < 0$ 1 minimum

For $c > 0$ 0 minimum

b) Volume.

Exponential damped.



$r = e^{-x}$, $A = \pi e^{-2x}$

a) what is the volume for $0 \le x \le 10$

b) What is the volume for infinite dim from 0 to ∞.

$$\pi \int_0^{10} e^{-2x}\,dx$$

$$\pi \frac{e^{-2x}}{(-2)} \Big|_0^{10} \quad (+)$$

$$= \boxed{\pi\left(\frac{1}{2} - \frac{e^{-20}}{2}\right)} \quad (+)$$

b) limit $t \to \infty$

$$\pi\left(\frac{1}{2} - \frac{e^{-\infty}}{2}\right) = \boxed{\frac{\pi}{2}}$$

c) $I = \int \cos 3x \sin 5x \, dx$

$= \frac{\sin 3x}{3} \sin 5x - \int \frac{\sin 3x}{3} 5\cos 5x \, dx$

$= \frac{\sin 3x}{3} \sin 5x + \frac{\cos 3x}{9} 5\cos 5x - \int \frac{\cos 3x}{9} 25 \sin 5x$

Solve for I $\quad \frac{25}{9} I$

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d) $\int (3x+2)^3 e^{5x} \, dx$

$(3x+2)^3$	e^{5x}	Sign
$3(3x+2)^2$	$e^{5x}/5$	+
$6(3x+2)$	$e^{5x}/25$	−
18	$e^{5x}/125$	+
0	$e^{5x}/625$	−

$$+ (3x+2)^3 \frac{e^{5x}}{5}$$

$$- 3(3x+2)^2 \, e^{5x}/25 \cdot 3$$

$$+ 6(3x+2) \, e^{5x}/125 \cdot 9$$

$$- 18 \, e^{5x}/625 + C$$