Unit 31 | Calc and Ec

I) Costs
II) Regression

Derivatives
Extremiz.

I)
1) Costs
2) Strawberry thm
3) Proof
4) Slides

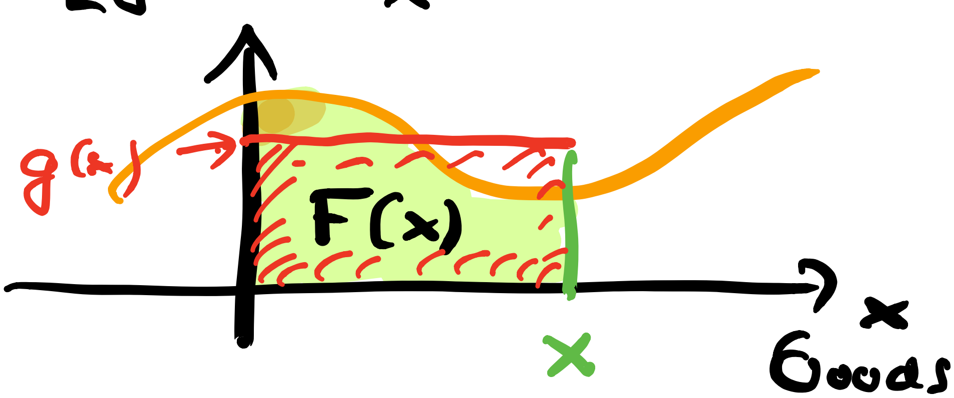
1) Jargon

x good

$f(x)$ marginal cost

$F(x)$ total cost $F' = f$

$g(x) = \frac{F(x)}{x}$ average cost



2) $\boxed{\text{Strawberry theorem}}$

Theorem

$$\boxed{g'(x) = 0 \text{ if and only if } f(x) = g(x)}$$

break even points.

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3) **Proof:**

$$g'(x) = \left(\frac{F(x)}{x}\right)' = \frac{x F' - F \cdot 1}{x^2} = \frac{1}{x}\left(F'(x) - \frac{F(x)}{x}\right)$$

$$= \frac{1}{x}(f - g) \quad \text{QED}$$

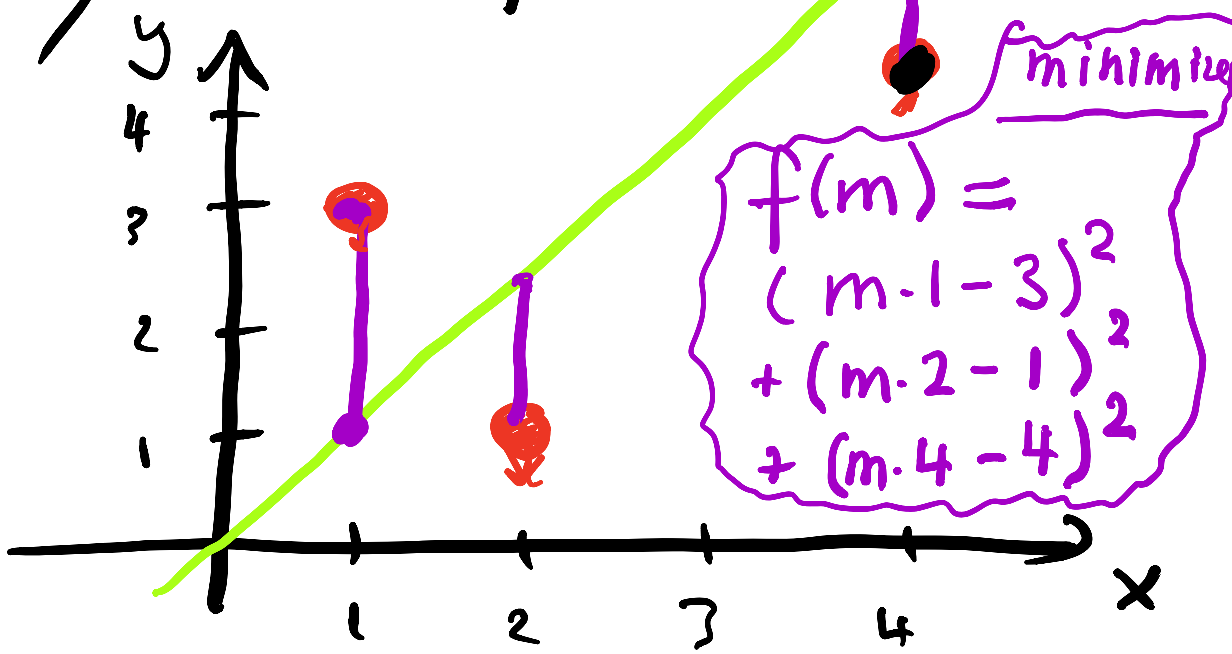
QED: quod erat demonstrandum

"Quite easily done"

Now to part II!

II $\boxed{\text{Regression}}$

1) Review of extremization



minimize

$$f(m) = (m\cdot 1 - 3)^2 + (m\cdot 2 - 1)^2 + (m\cdot 4 - 4)^2$$

__Regression problem:__ Find the line $y = mx + b$ which minimizes the sum of the squares of errors.

We look at the simpler problem of finding the best

$$y = mx$$

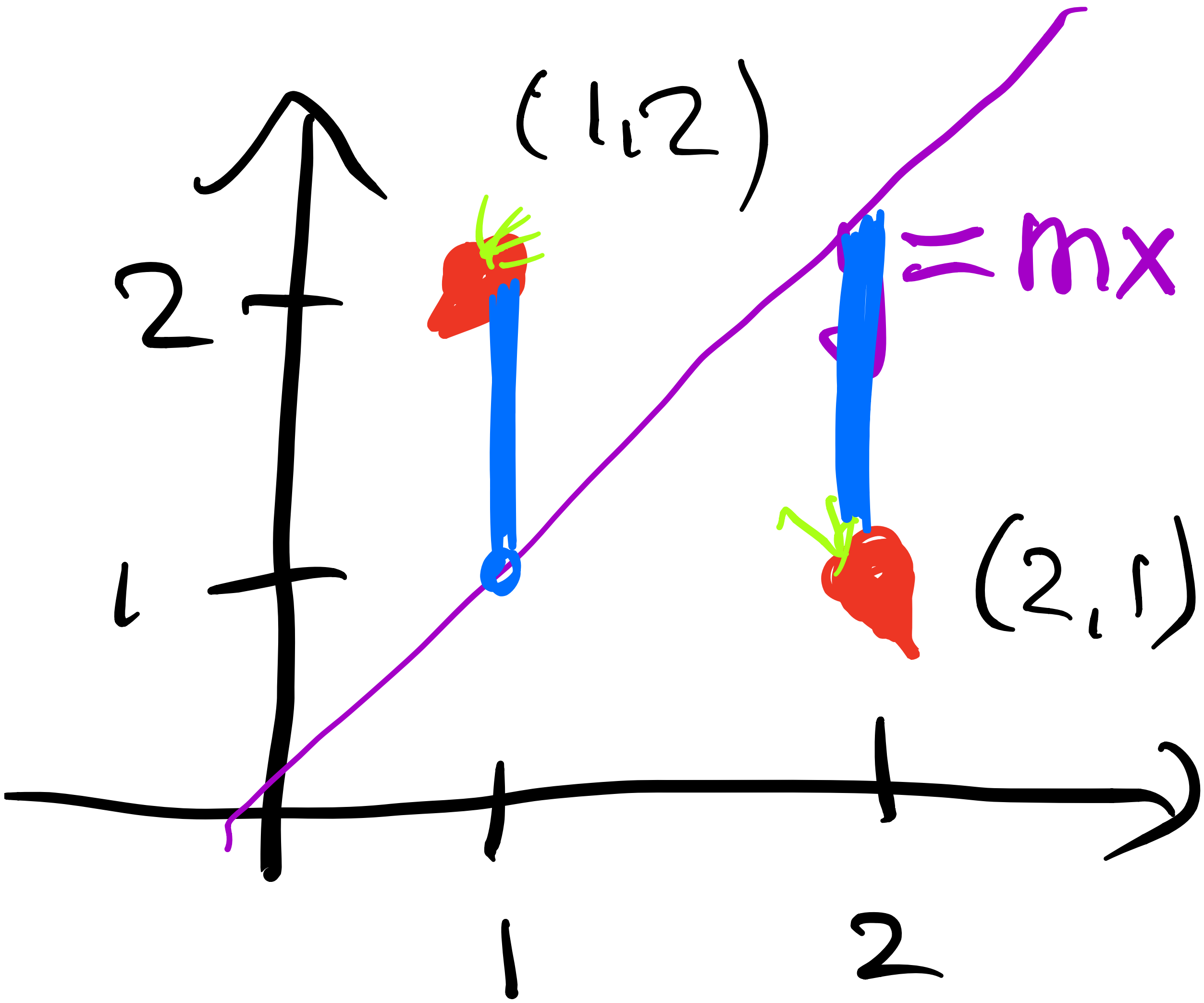
How do we find the minimal m?

$$f'(m) = 2(m\cdot 1 - 3)\cdot 1 + 2(m\cdot 2 - 1)\cdot 2 + 2(m\cdot 4 - 4)\cdot 4$$
$$= m(2 + 8 + 32) - (6 + 4 + 32)$$
$$= 0$$
$$m = \frac{42}{42} = 1$$

$$\boxed{\text{Fit}[\{\{1,3\},\{2,1\},\{4,4\}\}, \{x\}, x]}$$

Mathematica

2) Example



What is the best linear fit?

$$f(m) = (m-2)^2 + (2m-1)^2$$

$$f'(m) = 2(m-2) + 4(2m-1)$$

$$= 10m - 8 = 0$$

$$m = \boxed{\frac{4}{5}}$$

Conundrum!

Why not

$m = 1$

due to symmetry?