

INTRODUCTION TO CALCULUS

MATH 1A

UNIT 35: WORKSHEET

1: Name dropping: Match results with names

Result	Enter A-G
Fundamental theorem of trigonometry	B
Newton stpdf	E
Fundamental theorem of calculus	A
Mean value theorem	G
Rolle's theorem	F
Intermediate value theorem	C
Fermat theorem	D

A)	$\int_0^1 f'(x) dx = f(1) - f(0)$
B)	$\lim_{x \rightarrow 0} \sin(x)/x = 1$
C)	$f(0) = -1, f(1) = 1$ implies $f(x) = 0$ for some $x \in (0, 1)$.
D)	f is continuous on $[0, 1]$ then f has a global max and min on $[0, 1]$.
E)	$T(x) = x - f(x)/f'(x)$.
F)	If $f(0) = f(1) = 0$ then $f'(x) = 0$ for some $x \in (0, 1)$.
G)	There exists x in $(0, 1)$ such that $f'(x) = f(1) - f(0)$.

2: Match the following functions (a-d) with their derivatives (1-4) and second derivatives (A-D).

Function a)-d)	Fill in 1)-4)	Fill in A)-D)
graph a)	4	B
graph b)		
graph c)		
graph d)		

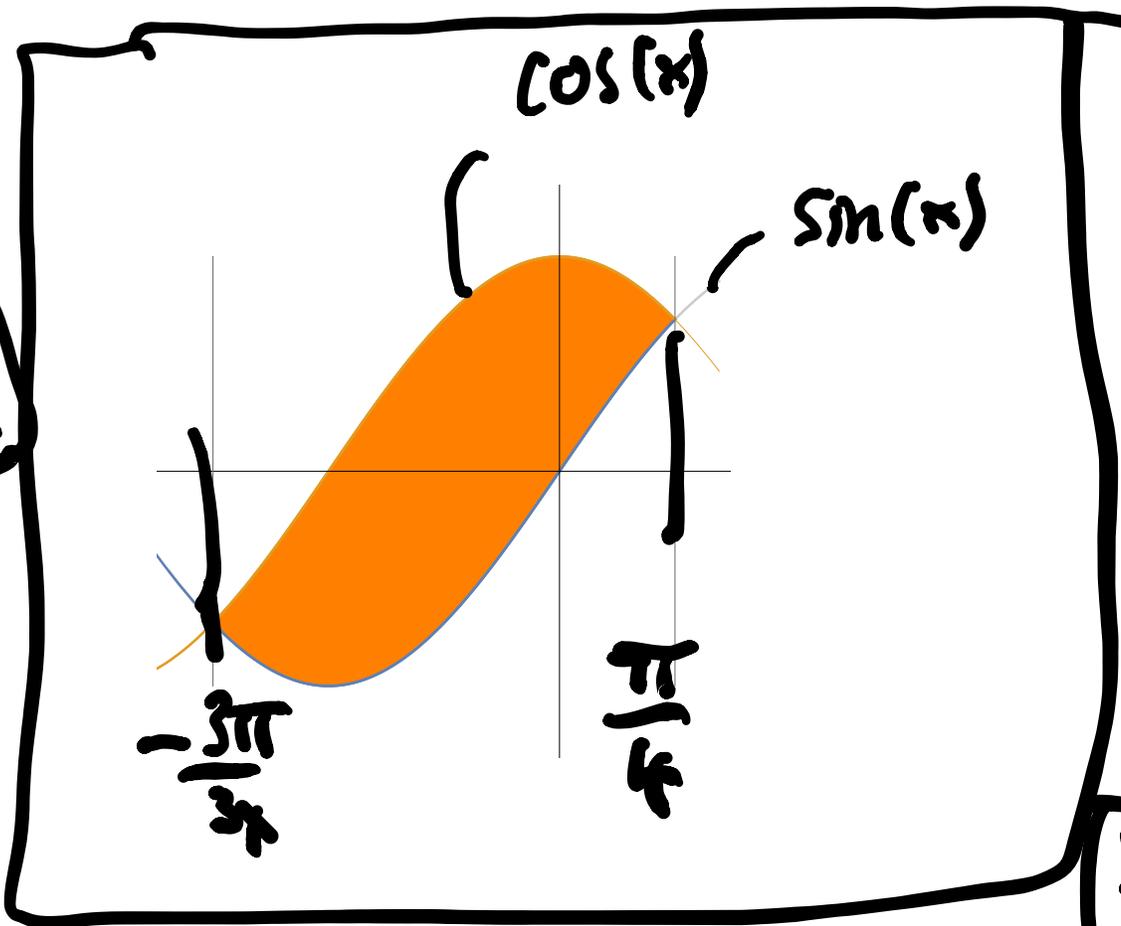
The image shows a matching exercise for calculus. It consists of a table at the top and a grid of graphs below. The table has three columns: "Function a)-d)", "Fill in 1)-4)", and "Fill in A)-D)". The first row of the table is filled with "4" and "B". Below the table are 12 graphs arranged in a 3x4 grid. The top row has graphs labeled a), b), c), and d). The middle row has graphs labeled 1), 2), 3), and 4). The bottom row has graphs labeled A), B), C), and D). Each graph shows a function and its derivative or second derivative. Handwritten marks include "X" and "3" above the first graph, and "X" above the third graph in the top row. The graphs are color-coded: a) is blue, b) is yellow, c) is yellow, d) is blue, 1) is blue, 2) is yellow, 3) is pink, 4) is green, A) is pink, B) is green, C) is blue, and D) is yellow.

3:

Find the area enclosed by the functions $f(x) = \sin(x)$ and $f(x) = \cos(x)$ and between the vertical lines $x = -3\pi/4$ and $x = \pi/4$.

$$\int_{-3\pi/4}^{\pi/4} (\cos x - \sin x) dx = \sin x + \cos x \Big|_{-3\pi/4}^{\pi/4}$$

Picture



$3/\sqrt{2}$

$$= \left(\sin\left(\frac{\pi}{4}\right) + \cos\left(\frac{\pi}{4}\right) - \sin\left(\frac{3\pi}{4}\right) - \cos\left(\frac{3\pi}{4}\right) \right) = \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}} - \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}}$$

4:

If we rotate the graph of the function $f(x) = \sqrt{\tan(x)}$ from $x = 0$ to $x = \pi/4$ we obtain a flower shaped solid. The two pictures below allow to admire it from two sides. Find its volume.

$\log\left(\frac{1}{x}\right) = -\log x$

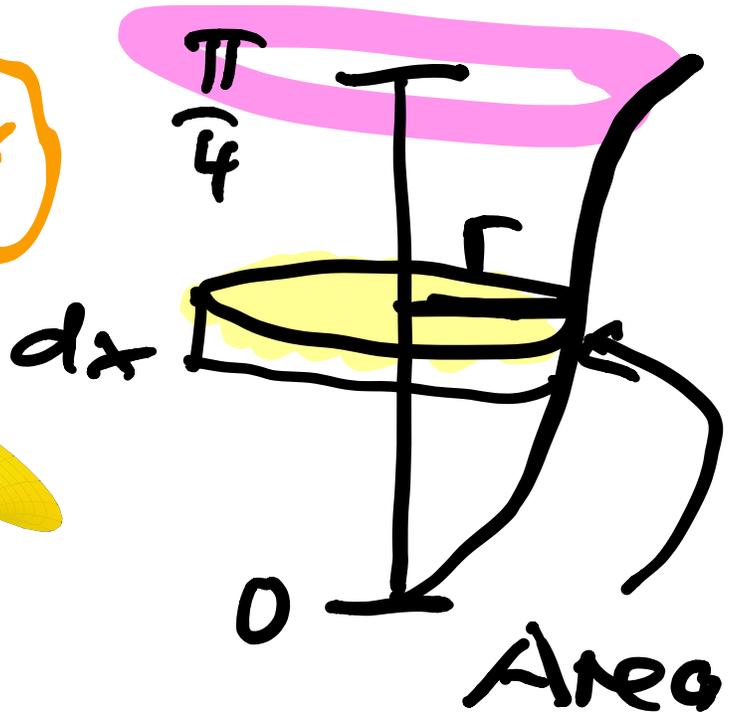
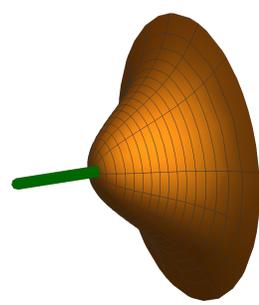
$\tan(x)$



$\frac{\sin x}{\cos x} \sim \frac{du}{dx}$

$\cos x$

u



$A(x)$

$= \pi \tan(x)$

$\pi/4$

$V = \pi \int_0^{\pi/4} \tan(x) dx =$

$\pi (-\log(\cos x)) \Big|_0^{\pi/4}$

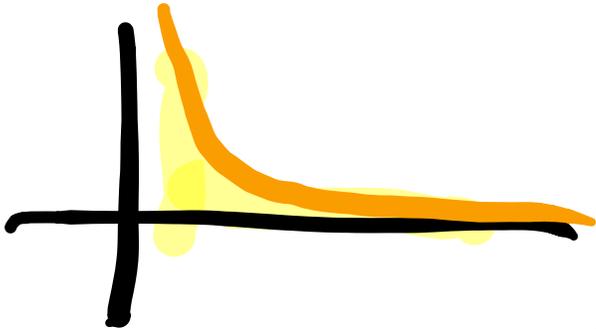
$$= \pi \left(-\log\left(\frac{1}{\sqrt{2}}\right) + \log(1) \right)$$

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$$= \boxed{\pi \log(\sqrt{2})}$$

5:

a) Find the integral or state that it does not exist



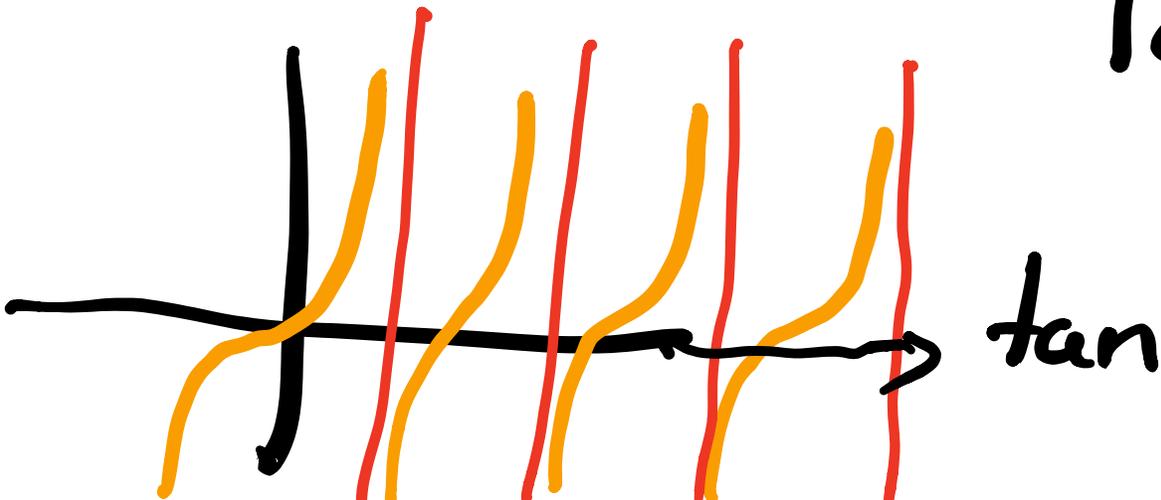
$$\int_1^{\infty} \frac{1}{x^5} dx = \int_1^{\infty} x^{-5} dx$$

$$= \frac{x^{-4}}{(-4)} \Big|_1^{\infty} = \boxed{\frac{1}{4}}$$

b) Find the integral or state that it does not exist

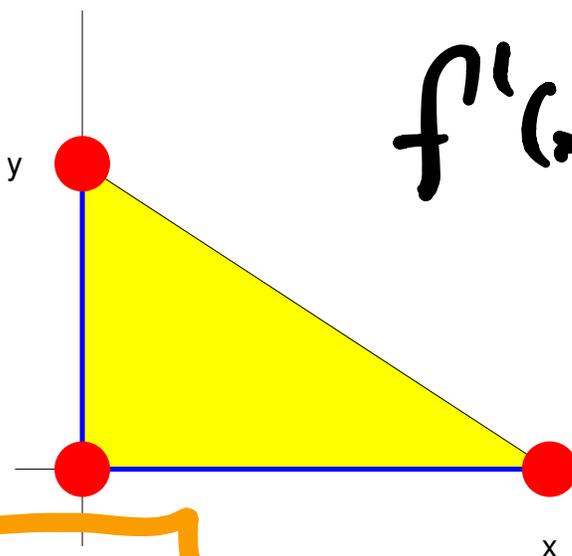
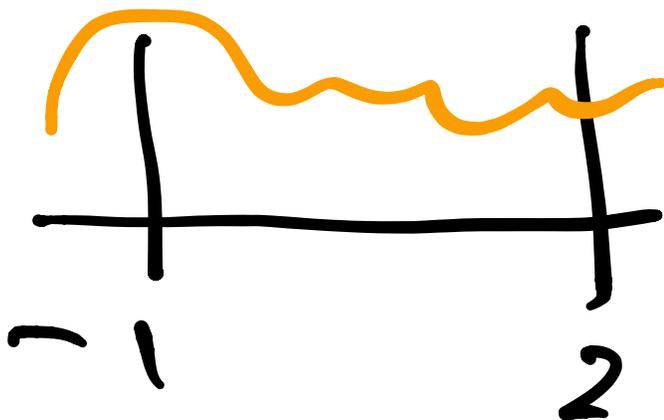
Does not converge.

$$\int_0^{\infty} \frac{1}{\cos^2(x)} dx = \tan(x) \Big|_0^{\infty}$$



6:

Find the local and global maxima of $f(x) = 2x^3 - 3x^2$ on the interval $[-1, 2]$. Use the second derivative test to check local extrema.



$$f'(x) = 6x^2$$

$$-6x = 0$$

$$= 6x(x-1)$$

$x = 0, x = 1$
are roots

$$f''(x) = 12x - 6 = \begin{cases} x=0 & -6 \\ x=1 & 6 \end{cases}$$

$x = 0$	max
$x = 1$	min

7:

Find the anti-derivative:

$$\begin{aligned}
 & -x^4 \cos(x-1) + 4x^3 \sin(x-1) \\
 & + 12x^2 \cos(x-1) \\
 & - 24x \sin(x-1) \\
 & - 24 \cos(x-1) + C
 \end{aligned}$$

$\int x^4 \sin(x-1) dx$

x^4	$\sin(x-1)$	Sign
$4x^3$	$-\cos(x-1)$	+
$12x^2$	$-\sin(x-1)$	-
$24x$	$\cos(x-1)$	+
24	$\sin(x-1)$	-
0	$-\cos(x-1)$	+

8:

a) Solve the integral $\int \sin^4(x) \cos(x) dx$.b) Solve the integral $\int \sqrt{1+2x} dx$.

a)

$$u = \sin x$$

$$du = \cos x dx$$

$$= \int u^4 du$$

$$= \int u^{1/2} \frac{du}{2}$$

b)

$$u = 1 + 2x$$

$$du = 2 \cdot dx$$

9:
Find

$$\int_4^5 \left(\frac{1}{x-2} - \frac{1}{x-3} \right) dx$$

$$= \log(x-2) - \log(x-3)$$

$$\int_4^5 \frac{1}{(x-2)(x-3)} dx.$$

$$\frac{1}{(x-2)(x-3)} = \frac{A}{x-2} + \frac{B}{x-3}$$

$$x = 2$$

$$-1 = A$$

$$\frac{1}{(x-2)(x-3)} = \frac{A}{x-2} + \frac{B}{x-3}$$

$$x = 3$$

$$A = 1$$

10:

Find the anti-derivatives of the following functions:

a) $f(x) = \log(x)/x.$

b) $f(x) = \frac{1}{x^2-4}.$

c) $f(x) = \frac{1}{x \log(x)}.$

Methods:

a) Sub! $u = \log x$
 $du = \frac{1}{x} dx$

b) $\frac{1}{x^2-4} = \frac{A}{x-2} + \frac{B}{x+2}$

c) $u = \log x$ $\int \frac{1}{u} du$
 $du = \frac{1}{x} dx = \log u$

$\log \log(x) + C$

11:

If $F(x) = \log(x)$ is the total cost and $f(x) = F'(x)$ is the marginal cost and $g(x) = \log(x)/x$ is the average cost:

a) Find the break-even point $f = g$.

b) Where is the average cost maximal?

c) Sweet surprise: why are the results in a) b) the same? The answer to c) is one word.

$$f = \frac{1}{x}$$

$$g = \frac{F}{x} = \frac{\log x}{x}$$

$$f = g \quad \therefore \quad \frac{1}{x} = \frac{\log x}{x}$$

$$\log x = 1, \quad x = e$$

