INTRODUCTION TO CALCULUS

MATH 1A

Unit 35: Worksheet

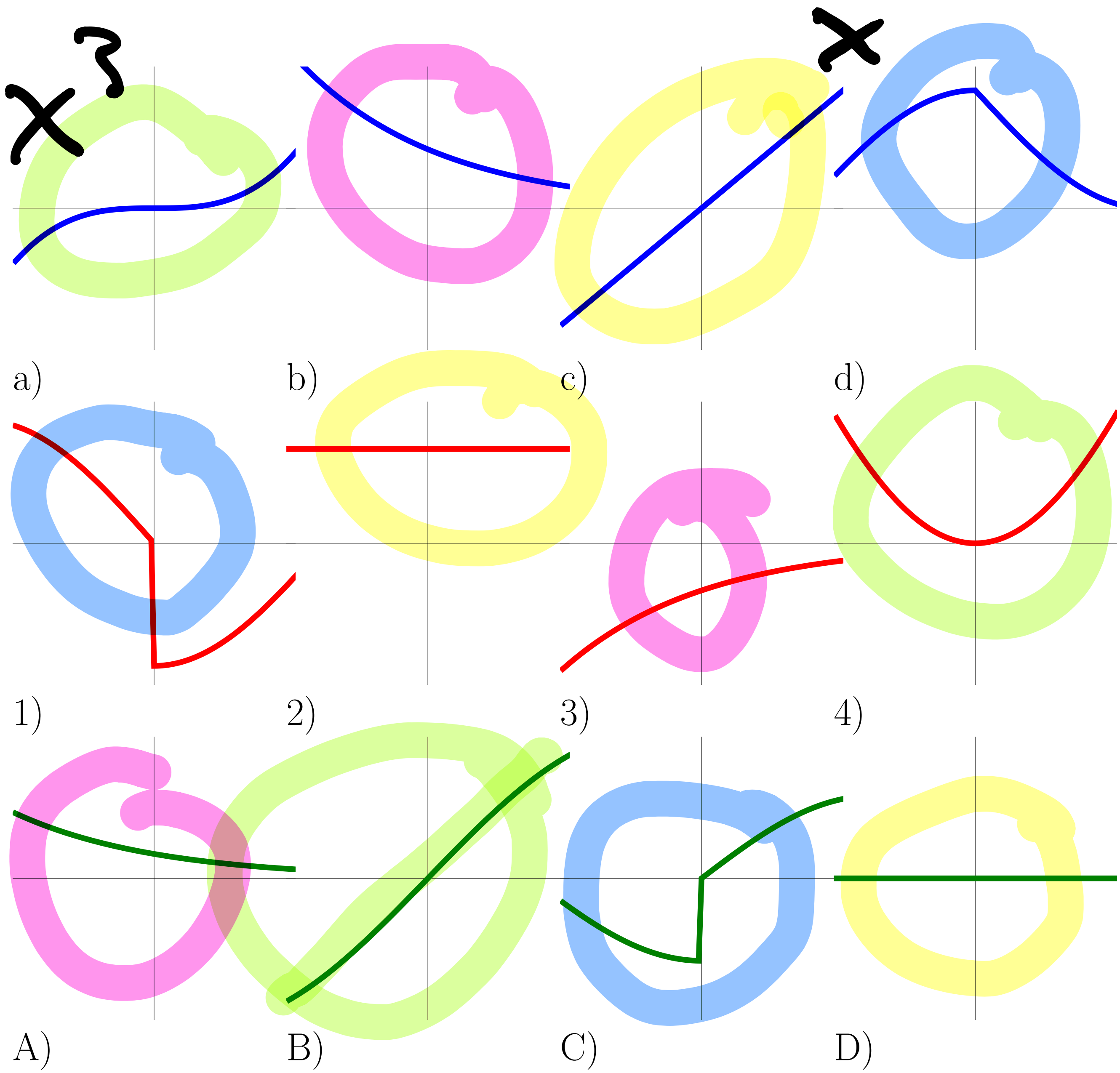
1: Name dropping: Match results with names

Result	Enter A-G
Fundamental theorem of trigonometry	B
Newton stpdf	E
Fundamental theorem of calculus	A
Mean value theorem	G
Rolle's theorem	F
Intermediate value theorem	C
Fermat theorem	D

A)	$\int_0^1 f'(x)\, dx = f(1) - f(0)$
B)	$\lim_{x\to 0} \sin(x)/x = 1$
C)	$f(0) = -1, f(1) = 1$ implies $f(x) = 0$ for some $x \in (0,1)$.
D)	f is continuous on $[0,1]$ then f has a global max and min on $[0,1]$.
E)	$T(x) = x - f(x)/f'(x)$.
F)	If $f(0) = f(1) = 0$ then $f'(x) = 0$ for some $x \in (0,1)$.
G)	There exists x in $(0,1)$ such that $f'(x) = f(1) - f(0)$.

2: Match the following functions (a-d) with their derivatives (1-4) and second derivatives (A-D).

Function a)-d)	Fill in 1)-4)	Fill in A)-D)
graph a)	4	B
graph b)		
graph c)		
graph d)		

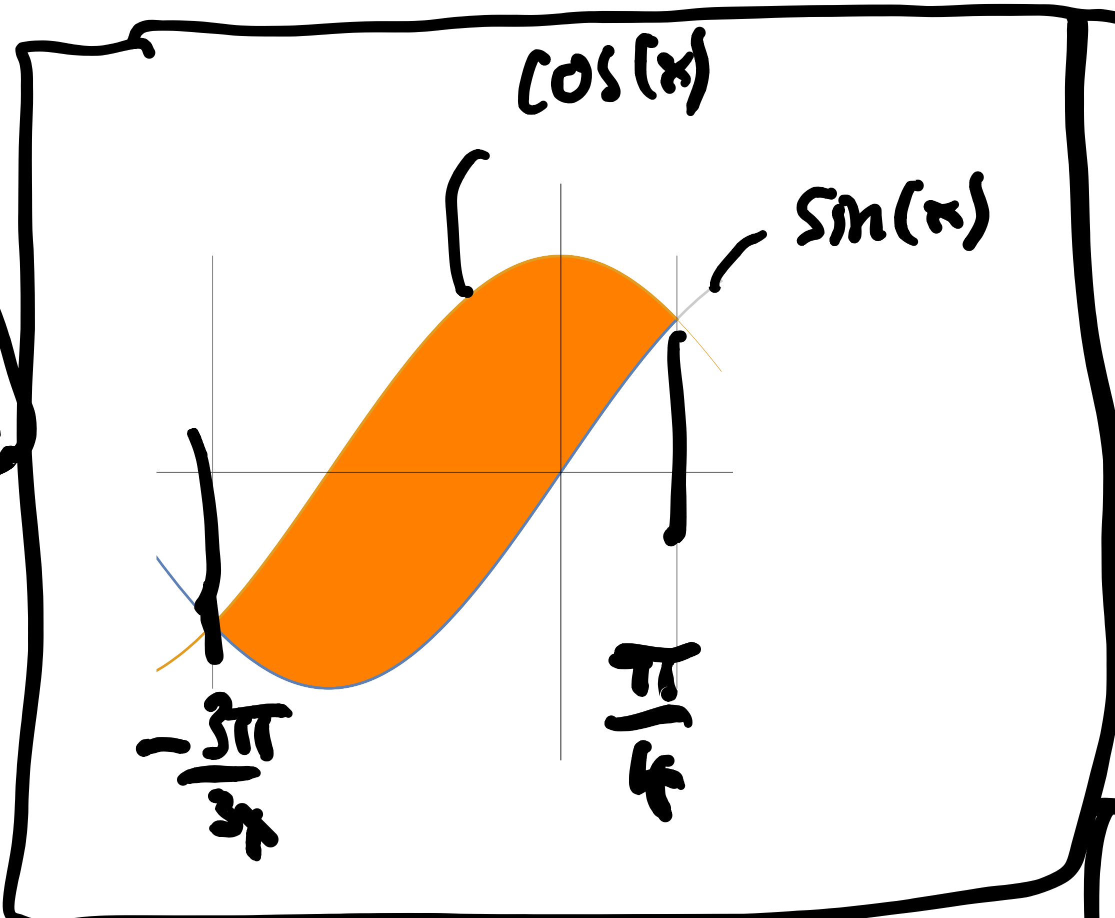


3:

Find the area enclosed by the functions $f(x) = \sin(x)$ and $f(x) = \cos(x)$ and between the vertical lines $x = -3\pi/4$ and $x = \pi/4$.

$$\int_{-\frac{3\pi}{4}}^{\pi/4} \cos x - \sin x \, dx = \sin x + \cos x \Big|_{-\frac{3\pi}{4}}^{\pi/4}$$

Picture



$$= \boxed{\sin\left(\frac{\pi}{4}\right) + \cos\left(\frac{\pi}{4}\right) - \sin\left(\frac{3\pi}{4}\right) - \cos\left(\frac{3\pi}{4}\right)} = \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}} - \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}} = \boxed{3/\sqrt{2}}$$

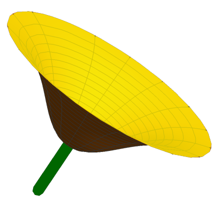
4:

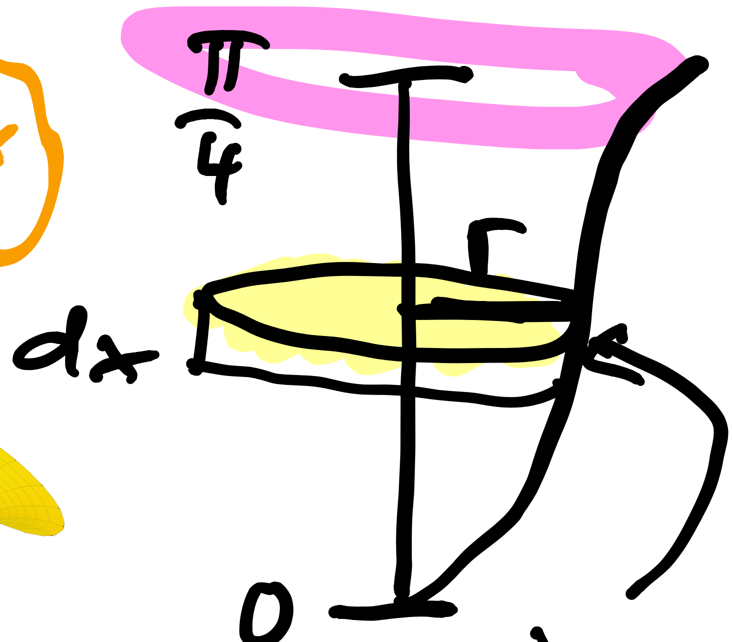
If we rotate the graph of the function $f(x) = \sqrt{\tan(x)}$ from $x = 0$ to $x = \pi/4$ we obtain a flower shaped solid. The two pictures below allow to admire it from two sides. Find its volume.

$\boxed{\log\left(\frac{1}{x}\right) = -\log x}$ (*)

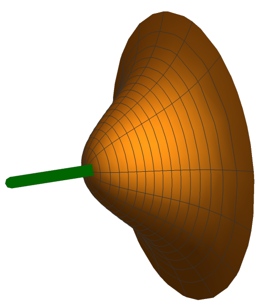
$$\tan(x) = \frac{\sin x}{\cos x}$$

$\sin x \sim -\frac{du}{dx}$, $\cos x$: u





Area



$$A(x) = \pi \tan(x)$$

$$V = \pi \int_0^{\pi/4} \tan(x)\, dx =$$

$$\pi\left(-\log(\cos x)\right)\Big|_0^{\pi/4}$$

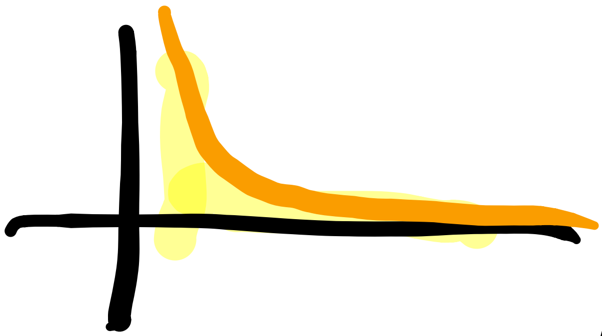
$$= \pi\left(-\log\left(\frac{1}{\sqrt{2}}\right) + \cancel{\log(1)}\right)$$

(*)

$$= \boxed{\pi \log(\sqrt{2})}$$

5:

a) Find the integral or state that it does not exist



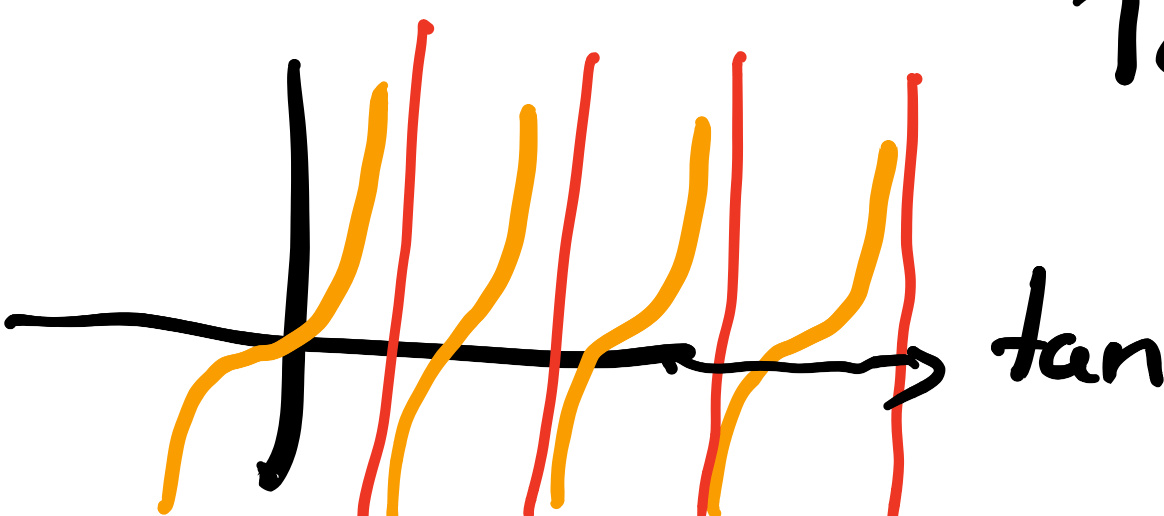
$$\int_1^\infty \frac{1}{x^5}\,dx = \int_1^\infty x^{-5}\,dx$$

$$= \frac{x^{-4}}{(-4)}\Big|_1^\infty = \boxed{\frac{1}{4}}$$

b) Find the integral or state that it does not exist

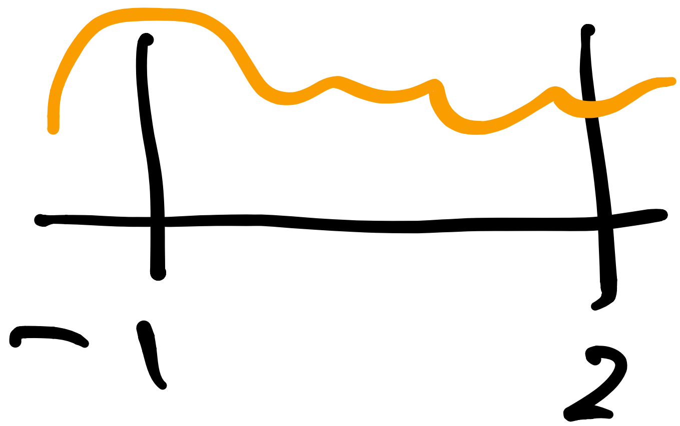
<u>Does not converge.</u>

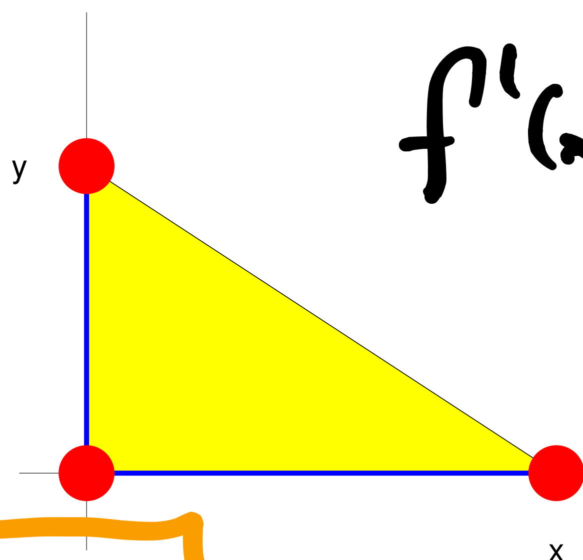
$$\int_0^\infty \frac{1}{\cos^2(x)}\,dx = \tan(x)\Big|_0^\infty$$



6:

Find the local and global maxima of $f(x) = 2x^3 - 3x^2$ on the interval $[-1, 2]$. Use the second derivative test to check local extrema.





$$f'(x) = 6x^2 - 6x = 0$$
$$= 6x(x-1)$$

$x=0$, $x=1$ are roots

$x=0$	max
$x=1$	min

$$f''(x) = 12x - 6 = \begin{cases} x=0 & -6 \\ x=1 & 6 \end{cases}$$

7:

Find the anti-derivative:

$$\int x^4 \sin(x-1)\, dx \,.$$

$$\boxed{-x^4\cos(x-1) + 4x^3\sin(x-1) + 12x^2\cos(x-1) - 24x\sin(x-1) - 24\cos(x-1) + C}$$

x^4	$\sin(x-1)$	Sign
$4x^3$	$-\cos(x-1)$	$+$
$12x^2$	$-\sin(x-1)$	$-$
$24x$	$\cos(x-1)$	$+$
24	$\sin(x-1)$	$-$
0	$-\cos(x-1)$	$+$

8:

a) Solve the integral $\int \sin^4(x)\cos(x)\, dx$. $= \int u^4\, du$

b) Solve the integral $\int \sqrt{1+2x}\, dx$. $= \int \frac{u^{1/2}\, du}{2}$

a) $u = \sin x$
$du = \cos x\, dx$

b) $u = 1+2x$
$du = 2 \cdot dx$

9:

Find

$$\int_4^5 \frac{1}{x-2} - \frac{1}{x-3}\,dx$$

$$\| \quad = \log(x-2) - \log(x-3)\Big|_4^5$$

$$\int_4^5 \frac{1}{(x-2)(x-3)}\,dx\,.$$

$$\frac{1}{\cancel{(x-2)}(x-3)}\cancel{(x-2)} = \frac{A}{\cancel{(x-2)}}\cancel{(x-2)} + \frac{B}{\cancel{(x-3)}}\cancel{(x-2)}$$

$$x = 2 \qquad -1 = A$$

$$\frac{1}{(x-2)\cancel{(x-3)}}\cancel{(x-3)} = \frac{A}{\cancel{(x-2)}}\cancel{(x-3)} + \frac{B}{\cancel{(x-3)}}\cancel{(x-3)}$$

$$x = 3 \qquad A = 1$$

10:
Find the anti-derivatives of the following functions:

a) $f(x) = \log(x)/x$.

b) $f(x) = \frac{1}{x^2-4}$.

c) $f(x) = \frac{1}{x\log(x)}$.

Methods:

a) Sub: $u = \log x$

$du = \frac{1}{x} dx$

b) $\frac{1}{x^2-4} = \frac{A}{x-2} + \frac{B}{x+2}$

c) $u = \log x$ $\quad \int \frac{1}{u} du$

$du = \frac{1}{x} dx \quad = \log u$

$\log \log(x) + C$

11:
If $F(x) = \log(x)$ is the total cost and $f(x) = F'(x)$ is the marginal cost and $g(x) = \log(x)/x$ is the average cost:.

a) Find the break-even point $f = g$.

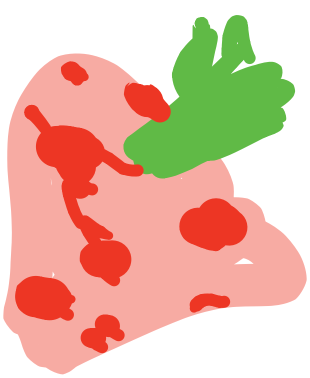
b) Where is the average cost maximal?

c) Sweet surprise: why are the results in a) b) the same? The answer to c) is one word.

$$f = \frac{1}{x}$$

$$g = \frac{F}{x} = \frac{\log x}{x}$$

$$f = g: \quad \frac{1}{x} = \frac{\log x}{x}$$



$$\log x = 1, \; x = e$$