MATH 1A - problem set 1

*PROBLEM 1.1

· predict the future and find the next term in the sequence

2		10		30		68		130		222		350		520		738		1010		1342		1740
	8		20		38		62		92		128		170		218		272		332		398	
		12		18		24		30		36		42		48		54		60		66		

first I started by searching for derivatives and then I noticed the difference between each derivative was 6. by following that sequence I integrated the missing term.

* PROBLEM 1.2

$f(n) = 2n+1$

$Sf(1) = 1$ $Sf(4) = 1+3+5+7 = 16$

$Sf(2) = 1+3 = 4$ $=>$ $Sf(5) = 1+3+5+7+9 = 25$ $=>$ $Sf(x) = x^2$ $Dg(x) = 2x$

$Sf(3) = 1+3+5 = 9$ $Sf(6) = 1+3+5+7+9+11 = 36$ $Sf(x) = g(n)$ $f(x) = 2x+1$

$Sf(7) = 1+3+5+7+9+11+13 = 49$ $g(x) = x^2$ $g(x+1) = (x+1)^2$

$= x^2 + 2x + 1$

$$\boxed{Dg(x) = g(x+1) - g(x) = f(x)}$$
$$Dg(x) = x^2 + 2x + 1 - x^2 = 2x+1$$
$$2x+1 = 2x+1$$

* PROBLEM 1.3

$f(x) = 2^x$ $Df(x) = f(x)$

$f(0) = 1$

$f(1) = 2$ $Df(x) = f(x+1) - f(x)$

$f(2) = 4$ $= 2^{x+1} - 2^x$

$= 2^x \cdot 2 - 2^x$

$= 2^x \cdot (2-1)$

$= 2^x \cdot 1$

$= 2^x$ $=>$ $\boxed{Df(x) = f(x)}$

* PROBLEM 1.4

PRIMES : 3 , 5 , 7 , 11 , 13 , 17 , 19 , 23 , 29 , 31 , 37 , 41 , 43 , 47 , 53 ,
59 , 61 , 67 , 71 , 73 , 79 , 83 , 89 , 97 , 101 , 103 , 107 , 109 ,

* PROBLEM 1.5

$g = Sf$
$Dg = f$

4) $f(x) = x + 1$, $g(x) = x(x+1)/2$

$$g(x) = \frac{x(x+1)}{2} = \frac{x^2 + x}{2}$$

$$Dg(x) = \frac{2 \cdot (2x+1) - (x^2+x) \cdot 0}{4}$$

$$Dg(x) = \frac{2(2x+1)}{4} = \frac{2x+1}{2}$$

$Dg(x) = g(x+1) - g(x) = f(x)$

$$Dg(x) = \frac{(x+1)(x+2)}{2} - \frac{x(x+1)}{2}$$

$$= \frac{x^2 + 2x + x + 2}{2} - \frac{x^2 + x}{2}$$

$$= \frac{x^2 + 2x + x + 2 - (x^2 + x)}{2}$$

$$= \frac{x^2 + 3x + 2 - x^2 - x}{2}$$

$$= \frac{2x + 2}{2} = \frac{2(x+1)}{2} = \boxed{x + 1}$$

B) $f(x) = x(x+1)$, $g(x) = x(x+1)(x+2)/6$

$$f(x) = \frac{(x+1)(x+2)}{2}$$

$$g(x) = \frac{x(x+1)(x+2)}{6}$$

$$g(x) = \frac{(x^2 + x)(x+2)}{6}$$

$$g(x) = \frac{x^3 + 2x^2 + x^2 + 2x}{6}$$

$$g(x) = \frac{x^3 + 3x^2 + 2x}{6}$$

$Dg(x) = g(x+1) - g(x) = f(x)$

$$Dg(x) = \frac{(x+1)(x+1+1)(x+1+2)}{6} - \frac{x(x+1)(x+2)}{6}$$

$$Dg(x) = \frac{(x+1)(x+2)(x+3)}{6} - \frac{x(x+1)(x+2)}{6}$$

$$Dg(x) = \frac{(x^2 + 2x + x + 2)(x+3)}{6} - \frac{(x^2+x)(x+2)}{6}$$

$$Dg(x) = \frac{(x^2 + 3x + 2)(x+3)}{6} - \frac{(x^2+x)(x+2)}{6}$$

$$Dg(x) = \frac{x^3 + 3x^2 + 9x + 2x + 6}{6}$$

$$- \frac{\left(x^3 + 2x^2 + x^2 + 2x\right)}{6}$$

$$Dg(x) = \frac{x^3 + 3x^2 + 11x + 6}{6}$$

$$\frac{- x^3 + 3x^2 + 2x}{6}$$

$$\frac{x^3 + 3x^2 + 11x + 6 - x^3 - 3x^2 - 2x}{}$$

$$= \frac{9x - 6}{6}$$

$$Dg(x) = g(x+1) - g(x) = f(x)$$

$$Dg(x) = \frac{(x+1)(x+2)(x+3)}{6} - \frac{x(x+1)(x+2)}{6}$$

$$= \frac{(x+1)(x+2)(x+3-x)}{6}$$

$$= \frac{(x^2 + 2x + x + 2)3}{6}$$

$$= \frac{x^2 + 3x + 2}{2}$$

$$= \frac{(x+1)(x+2)}{2}$$