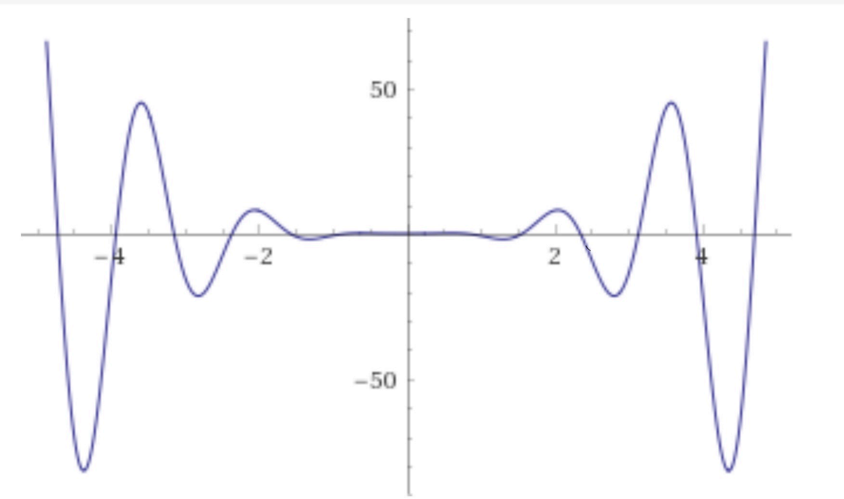
MATH 1A, PROBLEM SET 2 — Eva Galuza

* PROBLEM 2.1

$f(x) = x^3 \sin(4x) \qquad x \in [-5, 5]$



A) odd functions $f(-x) = -f(x)$

$x = 2$

$f(-2) \neq -f(2)$

$\Rightarrow$ function $f(x) = x^3 \sin(4x)$ $\boxed{\text{is not odd}}$

B) even functions $f(-x) = f(x)$ $\quad x = 2$

$f(x) = f(-x)$

$f(2) = f(-2)$

$\Rightarrow$ function $f(x) = x^3 \sin(4x)$ $\boxed{\text{is even}}$

C) For a function to be both even and odd at the same time it must fulfill the following condition $f(-x) = f(x) = -f(x)$ for all x in its domain. This is only possible if $f(x) = 0$

* PROBLEM 2.5

A) $f(x) = \cos(x)$

$\cos(x) = 0$

$x = \frac{1}{2}\pi$

$\boxed{a = \frac{1}{2}\pi}$

B) $f(x) = 4\exp(-x^4)$

$f(x) = 4e^{-x^4}$

$\Rightarrow$ exponential function

$\Rightarrow$ there is $\boxed{\text{no root}}$

C) $f(x) = x^5 - x^3$

$0 = x^5 - x^3$

$x^3 = x^5$

$x = 1$

$\boxed{a = 1}$

D) $f(x) = \log(x) = \ln(x)$

$0 = \log(x) = \ln(x)$

$x = \exp(0)$

$x = e^0$

$x = 1$

$\boxed{a = 1}$

E) $f(x) = \sin(x) - 1$

$0 = \sin(x) - 1$

$\sin(x) = 1$

$x = \frac{1}{2}\pi$

$\boxed{a = \frac{1}{2}\pi}$

F) $f(x) = \csc(x)$

$f(x) = \frac{1}{\sin x}$

there is $\boxed{\text{no root}}$

* PROBLEM 2.2 => horizontal and vertical line test to check whether the functions are invertible

A) $f(x) = x^7 - 1$ $A = R, B = R$

$y = x^7 - 1$

$x = y^7 - 1$

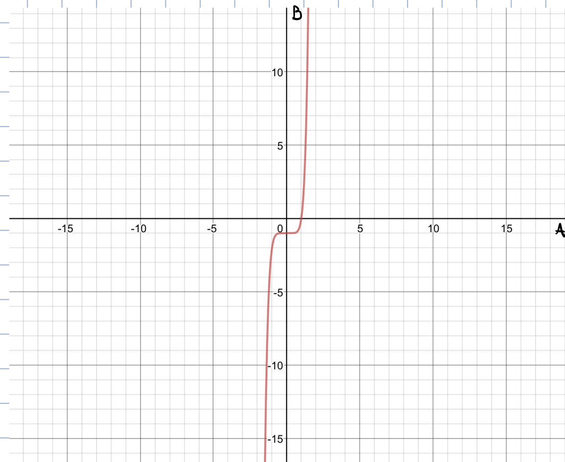
$-y^7 = -1 - x$ => this function is invertable

$y^7 = x + 1$

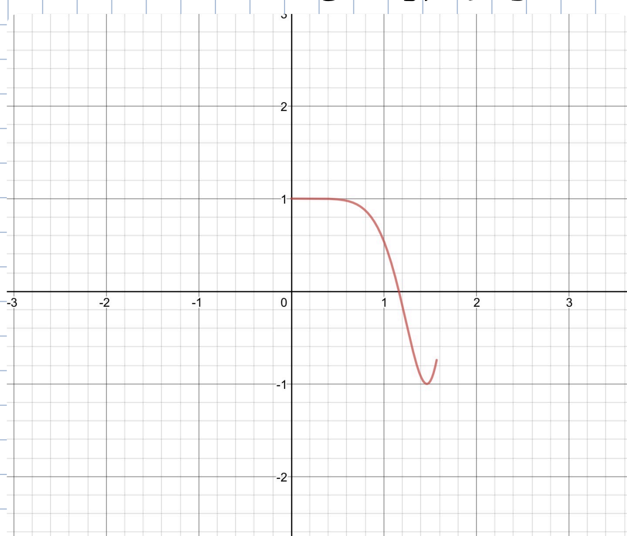
$y = \sqrt[7]{x+1}$

$y = (x+1)^{\frac{1}{7}}$

$\boxed{g(x) = (x+1)^{\frac{1}{7}}}$



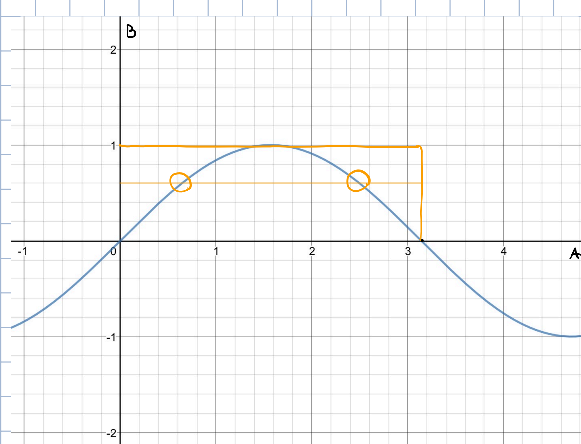
B) $f(x) = \cos(x^3)$ $A = [0, \pi/2]$, $B\,[-1, 1]$



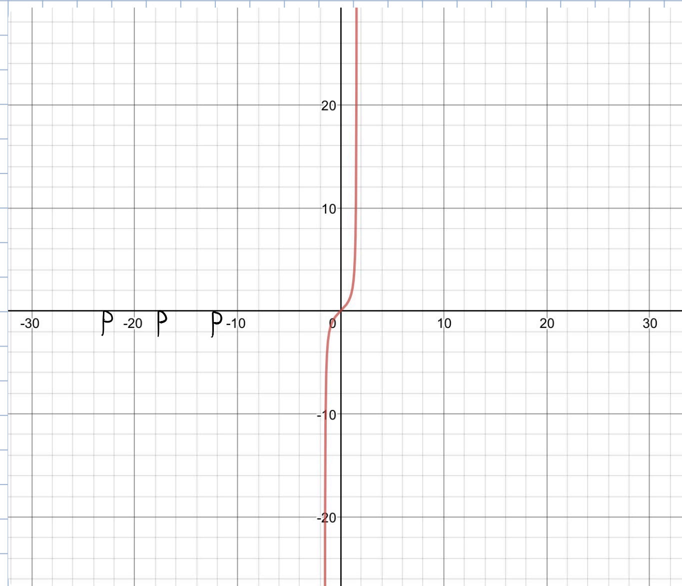
the function $f(x) = \cos(x^3)$ in this interval is not invertable

C) $f(x) = \sin(x)$ $A = [0, \pi]$, $B = [0, 1]$

=> $f(x) = \sin(x)$ is not invertible in the interval $A = [0, \pi]$ $B = [0, 1]$



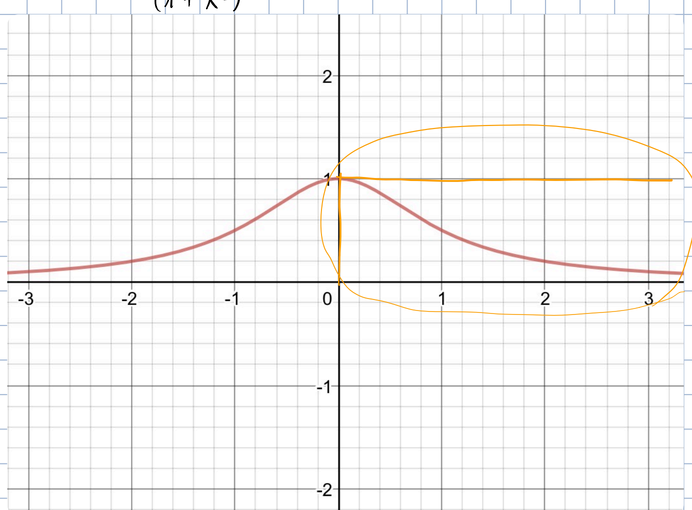
D) $f(x) = \tan(x) \quad A = (-\pi/2, \pi/2), \quad B = \mathbb{R}$



=> the function $f(x) = \tan(x)$ is invertible in the given intervale $A\ (-\pi/2, \pi/2),\ B \in \mathbb{R}$

$f(x) = \tan(x)$
$g(x) = \arctan(x)$

E) $f(x) = \frac{1}{(1+x^2)} \qquad A = [0, \infty), \quad B = (0, 1]$



the function $f(x) = \frac{1}{1+x^2}$ is invertible in the given intervale $A = [0, \infty), B = (0, 1]$

$f(x) = \frac{1}{1+x^2}$

$y = \frac{1}{1+x^2}$

$x = \frac{1}{1+y^2}$

$x(1+y^2) = 1$

$1 + y^2 = \frac{1}{x}$

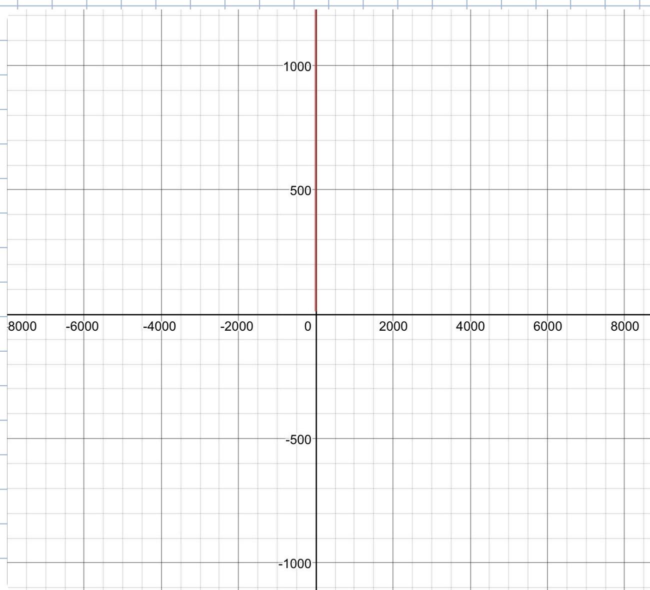
$y^2 = \frac{1}{x} - 1$

$y = \sqrt{\frac{1}{x} - 1}$

$\boxed{g(x) = \sqrt{\frac{1}{x} - 1}}$

* PROBLEM 2.4

$f(x) = \exp(\exp(\exp(x)))$ $x \in [0, 1]$

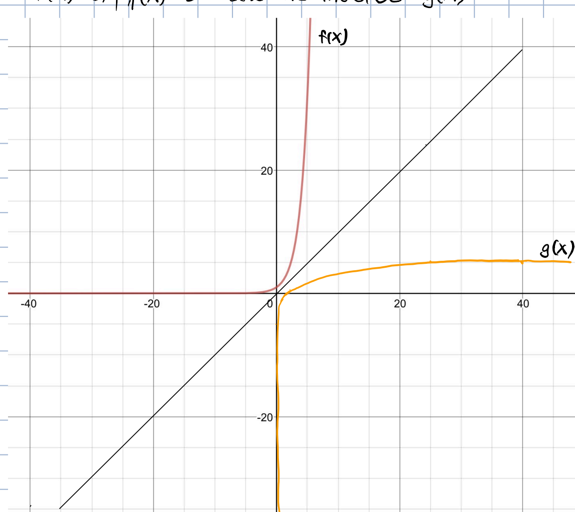


- When I ploted this function I noticed it grows very fast so it almost looks like a vertical line. It blows up really fast.

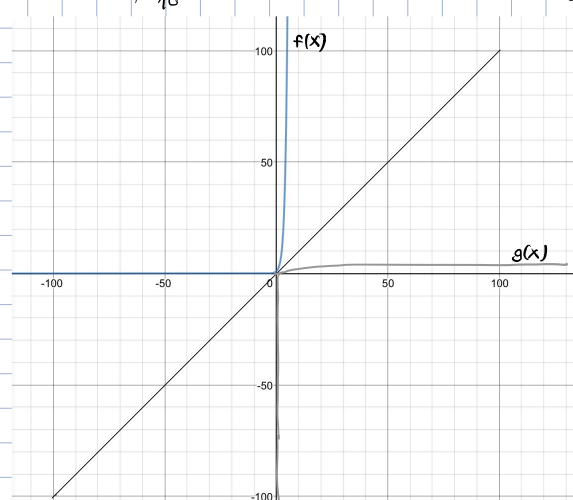
* PROBLEM 2.3

- $f(x) = \exp_1(x) = 2^x$
- $f(x) = \exp_{1/10}(x) = (1 + 1/10)^{10x}$
- $f(x) = \exp(x)$

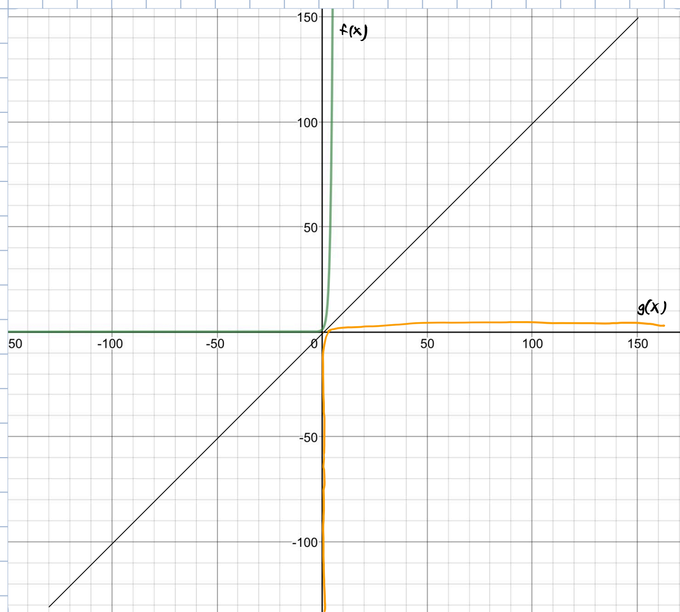
- $f(x) = \exp_1(x) = 2^x$ and its inverse $g(x)$



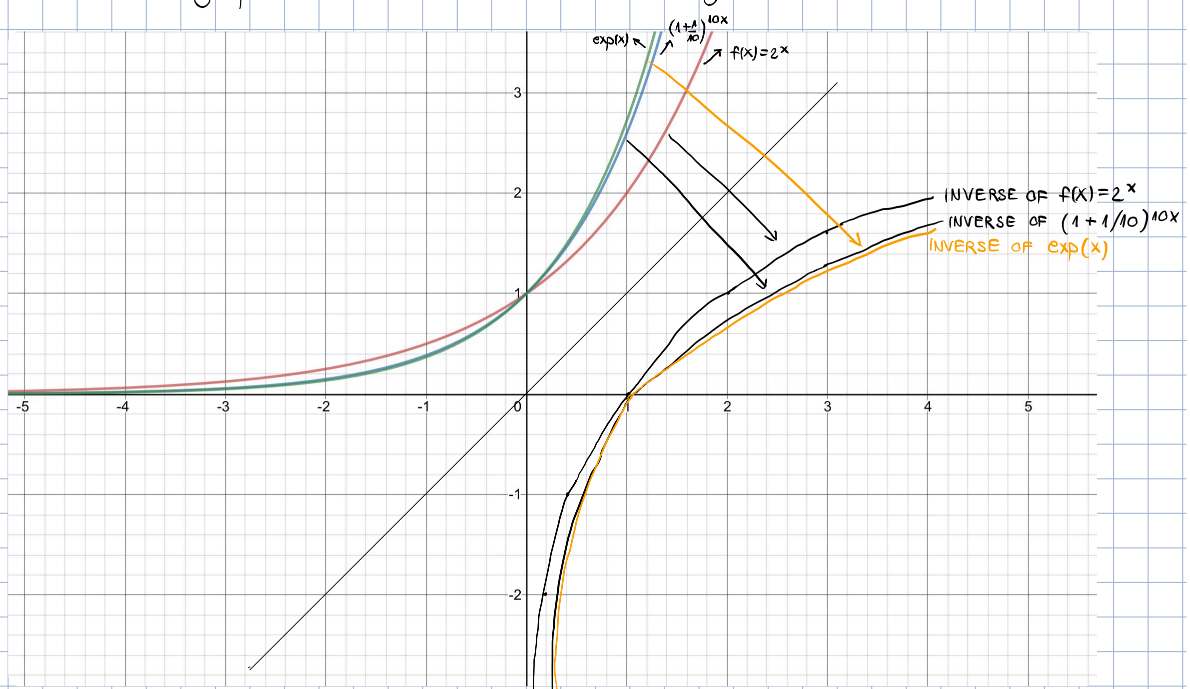
- $\exp_{\frac{1}{10}} x = (1 + 1/10)^{10x}$ and its inverse $g(x)$



• f(x)= exp(x) and its inverse g(x)



• here are all the graphs and their inverse functions together



⇒ I ACKNOWLEDGE THAT I USED „DESMOS" FOR THE GRAPHS