MATH 1A, PROBLEM SET 3 EVA GOLUZA

* PROBLEM 3.1

A) $\lim_{x \to 0} f(x) = \frac{(x^6-1)}{x-1} = \frac{0-1}{0-1} = \frac{-1}{1} = 1$ $\boxed{\lim_{x\to 0} \frac{(x^6-1)}{x-1} = 1}$

B) $\lim_{x\to 0} f(x) = \frac{\sin(23x)}{x}$ $\lim_{x\to 0} \frac{\sin(23x)}{x} \cdot \frac{23}{23} = \lim_{x\to 0} = \frac{\sin(23x)}{23x} \cdot 23 = 23$ $\boxed{\lim_{x\to 0} \frac{\sin(23x)}{x} = 23}$

C) $\lim_{x\to 0} f(x) = \frac{\sin^2(9x)}{x^2}$ $\lim_{x\to 0} \frac{\sin^2(9x)}{x^2} = \lim_{x\to 0} \frac{\sin(9x)}{x} \cdot \lim_{x\to 0} \frac{\sin(9x)}{x}$

$= \lim_{x\to 0} \frac{\sin(9x)}{x} \cdot \frac{9}{9} \cdot \lim_{x\to 0} \frac{\sin(9x)}{x} \cdot \frac{9}{9} = \lim_{x\to 0} \frac{\sin(9x)}{9x} \cdot 9 \cdot \lim_{x\to 0} \frac{\sin(9x)}{9x} \cdot 9 = 9 \cdot 1 \cdot 9 \cdot 1 = \boxed{81}$

D) $\lim_{x\to 0} f(x) = \frac{\sin(11x)}{\sin(7x)}$ $\lim_{x\to 0} \frac{\sin(11x)}{\sin(7x)} = \lim_{x\to 0} \underbrace{\frac{\sin(11x)}{1}}_{11} \cdot \lim_{x\to 0} \underbrace{\frac{1}{\sin(7x)}}_{1/7} = \frac{11}{7}$ $\boxed{\lim_{x\to 0} \frac{\sin(11x)}{\sin(7x)} = \frac{11}{7}}$

* PROBLEM 3.4

A) $\lim_{x\to 0} f(x) = \frac{(x^2-2x+1)}{(x-1)} = \frac{(x-1)^{\cancel{2}}}{(\cancel{x-1})} = (x-1) = 0-1 = -1$ $\boxed{\lim_{x\to 0} \frac{(x^2-2x+1)}{(x-1)} = -1}$

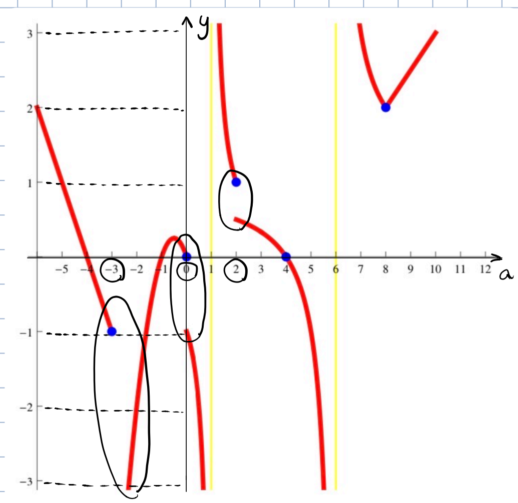
B) $\lim_{x\to 0} f(x) = \frac{\sin(x)}{x} \cdot \frac{2^x}{5^x}$ $\lim_{x\to 0} \frac{\sin(x)}{x} \cdot \frac{2^x}{5^x} = \lim_{x\to 0} \underbrace{\frac{\sin(x)}{x}}_{1} \cdot \lim_{x\to 0} \underbrace{\frac{2^x}{5^x}}_{1} = 1 \cdot 1 = 1$

$\boxed{\lim_{x\to 0} \frac{\sin(x)}{x} \frac{2^x}{5^x} = 1}$

C) $\lim_{x\to 0} f(x) = 2^{(2^{2^x})}$ $\lim_{x\to 0} 2^{2^{2^0=1}} = 2^2 = 4$ $\boxed{\lim_{x\to 0} 2^{(2^{2x})} = 4}$

D) $\lim_{x\to 0} f(x) = \frac{\sin(\sin(x))}{\sin(x)}$ $\lim_{x\to 0} \frac{\sin(\sin(x))}{\sin(x)} = \lim_{x\to 0} \underbrace{\frac{\sin(\sin(x))}{x}}_{1} \cdot \underbrace{\frac{x}{\sin(x)}}_{1} = 1$ $\boxed{\lim_{x\to 0} \frac{\sin(\sin(x))}{\sin x} = 1}$

* PROBLEM 3.3



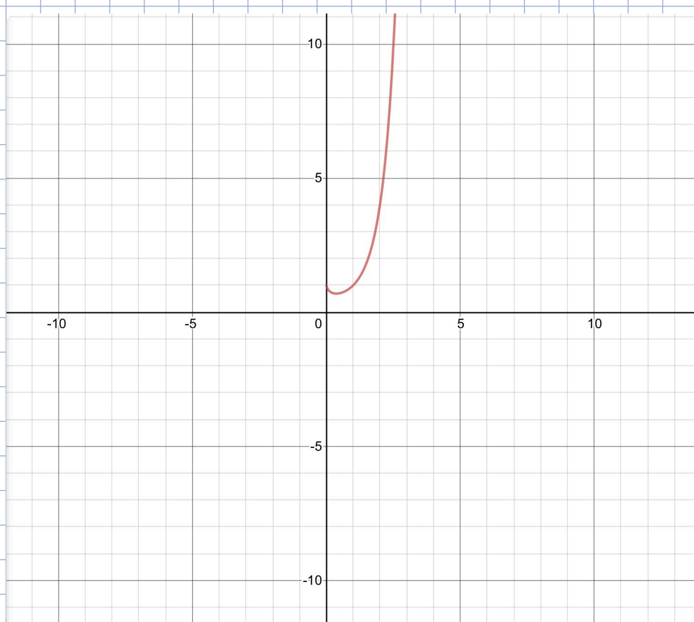
Points at which the function f(x) has no limits.

a=0	a=6
a=2	a=1
a=-3	

$$\left.\begin{array}{l}\lim\limits_{x\to 0^+} f(x) = a \\ \\ \lim\limits_{x\to 0} f(x) = b\end{array}\right\} \; = \Rightarrow \text{there is a limit}$$

if $a \neq b \Rightarrow$ there is no limit

* PROBLEM 3.5



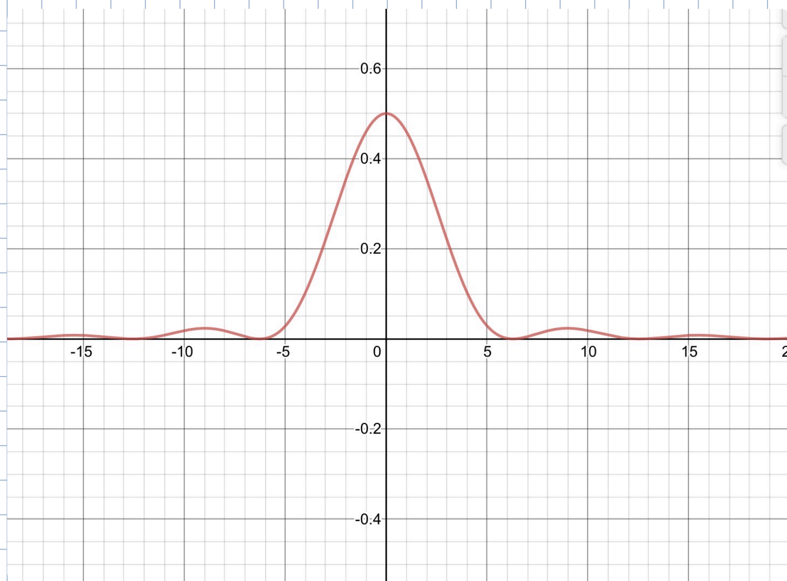
- $f(x) = x^x$

The function $f(x) = x^x$ is not defined for the values $x < 0$. When I graphed this function on desmos we can see that as x approaches 0 ($x \to 0$) from the right. Left from x=0 the function is not defined. When we use values that are extremly close to x=0 is a good method to find limits, because we cannot use the method of substition or visually determine it. I see that for x=0 it converges to 1.

*** PROBLEM 3.2**

$$f(x) = \frac{(1-\cos x)}{x^2}$$

A)



$$\boxed{\text{THIS FUNCTION IS NOT DEFINED AT } x=0}$$

B) $$\lim_{x\to 0} \frac{(1-\cos x)}{x^2} = \lim_{x\to 0} \frac{2\sin x^2\left(\frac{x}{2}\right)}{x^2} = \lim_{x \to 0} \frac{2\sin x^2\left(\frac{x}{2}\right)}{4\left(\frac{x}{2}\right)^2} = \frac{1}{2}$$

$$\cos(2x) = 1 - 2\sin^2 x$$

$$\Rightarrow \cos(x) = 1 - 2\sin^2\left(\frac{x}{2}\right)$$

$$\boxed{\lim_{x\to 0} \frac{(1-\cos x)}{x^2} = \frac{1}{2}}$$

C) $$f(x) = \frac{f(x+h)-f(x)}{h} = \left[\frac{(1+h)^{\frac{x+h}{h}} - (1+h)^{x/h}}{h}\right]$$

$$= \frac{(1+h)^{\frac{x}{h}} \cdot (1+h)^{\frac{h}{h}} - (1+h)^{\frac{x}{h}}}{h}$$

$$= \frac{(1+h)^{\frac{x}{h}}\left[(1+h) - 1\right]}{h}$$

$$= \frac{(1+h)^{\frac{x}{h}}(1+h-1)}{h}$$

$$= \frac{(1+h)^{\frac{x}{h}} \cdot \cancel{h}}{\cancel{h}} = \boxed{(1+h)^{\frac{x}{h}}} \Rightarrow \boxed{\left[f(x+h) - f(x)\right]/h = f(x)}$$