EVA GOLUZA

MATH 1.A – PROBLEM SET 4.

* PROBLEM 4.1

A) $\text{sinc}(x) + \frac{1}{\cos x} = \frac{\sin x}{x} + \frac{1}{\cos x}$

$\frac{\sin x}{x}$ ↓ CONTINOUS

$\cos(x) = 0$

$x = \frac{\pi}{2} + k\pi$

THE FUNCTION $f(x) = \frac{\sin x}{x} + \frac{1}{\cos x}$ IS NOT CONTINOUS FOR $x = \pi/2 + k\pi$. THE FUNCTION IS NOT CONTINOUS DUE TO A POLE.

B) $\sin(\tan(x))$

↓ CONTINOUS

$\tan(x)$ – UNDEFINED AT $x = \pi/2 + k\pi$

THE FUNCTION $f(x) = \sin(\tan(x))$ IS NOT CONTINOUS FOR $x = \pi/2 + k\pi$. THE FUNCTION IS NOT CONTINOUS DUE TO OSCILLATION.

C) $f(x) = \cot(2-x)$

$2 - x = 0$

$-x = -2$

$x = 2 + k\pi$

THE FUNCTION $f(x) = \cot(2-x)$ IS NOT CONTINOUS FOR $x = 2 + k\pi$. THE FUNCTION IS NOT CONTINOUS DUE TO A POLE

D) $\text{sign}(x)/x$

$\oplus \ \frac{1}{x}$

$\ominus \ -\frac{1}{x}$

$x = 0$

THE FUNCTION $f(x) = \text{sign}(x)/x$ IS NOT CONTINOUS FOR $x=0$. THE FUNCTION IS NOT CONTINOUS DUE TO THE ESCAPE TO INFINITY.

E) $\frac{x^2 + 5x + x^4}{x-3}$

$x - 3 = 0$

$x = 3$

THE FUNCTION $f(x) = \frac{x^2 + 5x + x^4}{x-3}$ IS NOT CONTINOUS FOR $x=3$. THE FUNCTION IS NOT CONTINOUS DUE TO A POLE

* PROBLEM 4.2

A) THE FUNCTION IS CONTINOUS AT THE FOLLOWING INTERVALS: $x \in (-2, 1)$, $x \in (1, 2)$

B) THE FUNCTION IS CONTINOUS AT THE FOLLOWING INTERVALS: $x \in (-6, -4)$, $x \in [-4, 1)$, $x \in [1, 2)$

* PROBLEM 4.4

A) $$\frac{(x^3-8)}{x-2} = \frac{\cancel{(x-2)}(x^2+2x+4)}{\cancel{x-2}} = \boxed{x^2+2x+4}$$

B) $$\frac{(x^5+x^3)}{(x^2+3)} \Rightarrow$$ THIS FUNCTION DOES NOT HAVE TO BE HEALED BECAUSE x^2+3 WILL ALWAYS BE POSITIVE AND THEREFORE THE DENOMINATOR CAN NEVER BE ZERO.

$\sin^2 x + \cos^2 x = 1$

C) $$\frac{((\sin(x))^3 - \sin(x))}{\cos(x)\sin(x)} = \frac{\sin(x)\cdot(\sin(x)^2 - 1)}{\cos(x)\sin(x)} = \frac{-\sin x(-\sin(x)^2+1)}{\cos(x)\sin(x)} = \frac{-\sin(x)\cdot(1-\sin(x)^2)}{\cos(x)\sin(x)}$$

$$= \frac{-\cancel{\sin(x)}\cdot\cos(x)^2}{\cos(x)\cancel{\sin(x)}} = \frac{-\cos(x)^{\cancel{2}}}{\cancel{\cos(x)}} = \boxed{-\cos(x)}$$

D) $$\frac{(x^4+4x^3+6x^2+4x+1)}{(x^3+3x^2+3x+1)} = \frac{x^4+4x^3+6x^2+4x+1}{(x+1)^3}$$

$$= \frac{x^4+x^3+3x^3+3x^2+3x^2+3x+x+1}{(x+1)^3}$$

$$= \frac{x^3\cdot(x+1)+3x^2(x+1)+3x(x+1)+1(x+1)}{(x+1)^3}$$

$$= \frac{\cancel{(x+1)}(x^3+3x^2+3x+1)}{(x+1)^{\cancel{3}2}}$$

$$= \frac{(x^3+3x^2+3x+1)}{(x+1)^2}$$

$$= \frac{(x+1)^{\cancel{3}1}}{\cancel{(x+1)^2}}$$

$$\boxed{= x+1}$$

E) $$\frac{(x^{70}-1)}{(x^{10}-1)} = \frac{(u^7-1)}{(u-1)} = \frac{\cancel{(u-1)}(u^6+u^5+u^4+u^3+u^2+u+1)}{\cancel{(u-1)}}$$

$x^{10} = u$

$$= u^6+u^5+u^4+u^3+u^2+u+1$$

$$\boxed{= x^{60}+x^{50}+x^{40}+x^{30}+x^{20}+x^{10}+1}$$

* PROBLEM 4.5

A) $\sin\left(\frac{1}{3+\sin(x)\cos(x)}\right) + |\cos(x)| + \frac{\sin(x)}{x} + x^5 + x^3 + 1 + \frac{7}{\exp(x)}$

- $\sin\left(\frac{1}{3+\sin(x)\cos(x)}\right)$ => THIS IS A CONTINOUS FUNCTION
- $|\cos(x)|$ => THIS IS A CONTINOUS FUNCTION
- $\frac{\sin(x)}{x}$ => THIS IS A CONTINOUS FUNCTION
- x^5 => THIS IS A CONTINOUS FUNCTION
- x^3 => THIS IS A CONTINOUS FUNCTION
- 1 => THIS IS A CONTINOUS FUNCTION
- $\frac{7}{\exp(x)}$ => THIS IS A CONTINOUS FUNCTION

SINCE ALL THESE PARTS ARE CONTINOS WE CAN CONCLUDE THAT THE FUNCTION

$f(x) = \sin\left(\frac{1}{3+\sin(x)\cos(x)}\right) + |\cos(x)| + \frac{\sin(x)}{x} + x^5 + x^3 + 1 + \frac{7}{\exp(x)}$ IS CONTINOUS

B) $\frac{2}{\log|x|} + x^7 - \cos(\sin(\cos(x))) - \exp(\log(\exp(x)))$

- $\frac{2}{\log|x|}$ => THIS FUNCTION IS NOT CONTINOUS
- x^7 => THIS FUNCTION IS CONTINOUS
- $\cos(\sin(\cos(x)))$ => THIS FUNCTION IS CONTINOUS
- $\exp(\log(\underbrace{\exp(x)}_{x}))$ => THIS FUNCTION IS CONTINOUS

SINCE $\frac{2}{\log|x|}$ IS NOT CONTINOUS WE CAN CONCLUDE THAT THE FUNCTION

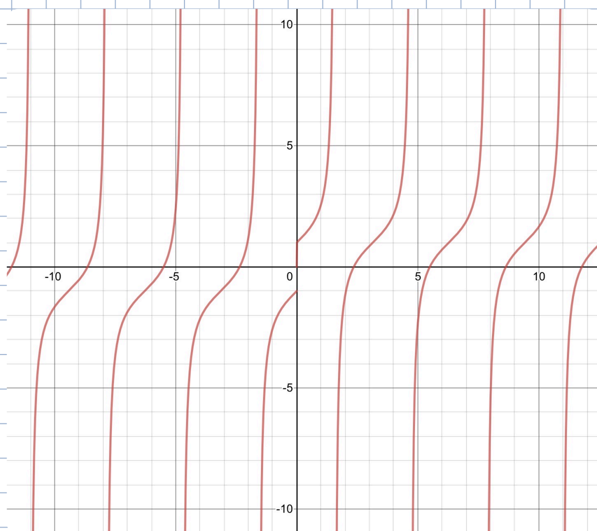
$f(x) = \frac{2}{\log|x|} + x^7 - \cos(\sin(\cos(x))) - \exp(\log(\exp(x)))$

IS NOT CONTINOUS

* PROBLEM 4.3

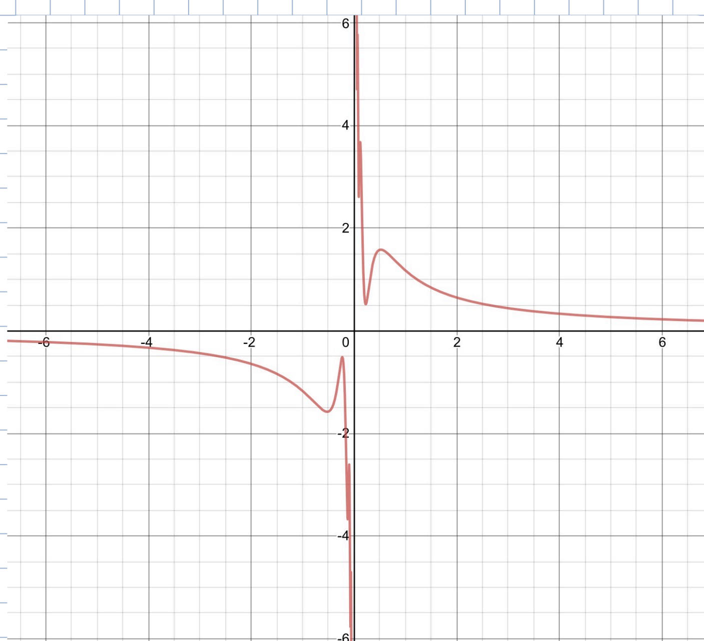
A) A FUNCTION THAT HAS A JUMP DISCONTINUITY AND AN ESCAPE TO INFINITY

$$\boxed{f(x) = \tan(x) + \operatorname{sign}(x)}$$



B) A FUNCTION WHICH HAS AN OSCILLATORY DISCONTINUITY AND AN ESCAPE TO INFINITY

$$\boxed{f(x) = \sin\left(\frac{1}{x}\right) + \frac{1}{3x}}$$



C) A FUNCTION WHICH HAS A JUMP DISCONTINUITY AS WELL AS AN OSCILLATORY DISCONTINUITY

$$\boxed{f(x) = \sin\left(\frac{1}{x}\right) + \text{sign}(x+2)}$$

* A FUNCTION WHICH HAS A JUMP DISCONTINUITY, A POLE, AND AN OSCICLATORY DISCONTINUITY ALL AT THE SAME TIME.

$$\boxed{f(x) = \frac{1}{\cos(x)} + \sin\left(\frac{1}{x}\right) + \text{sign}(x+2)}$$