MATH 1A - PROBLEM SET 5.

* PROBLEM 5.1

$f(x) = -x^5 - 3x^4 + 42x^3 + 62x^2 - 297x - 315$

$f(x) = -(x+1)(x-3)(x+3)(x-5)(x+7)$

$315 = 315 \cdot 1$
$315 = (-315) \cdot (-1)$
$315 = 45 \cdot 7$
$315 = (-45) \cdot (-7)$
$315 = 105 \cdot 3$
$315 = (-105) \cdot (-3)$
$315 = 63 \cdot 5$
$315 = (-63) \cdot (-5)$

NOW AFTER WE HAVE FOUND FACTORS OF 315, WE CAN PLUG THEM INTO THE FORMULA. THE ONES THAT END UP GIVING THE VALUE EQUAL TO ZERO ARE THE ROOTS.

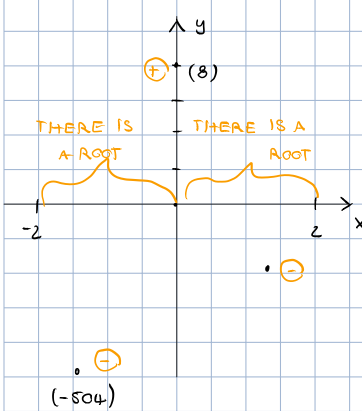
=>

THE ROOTS OF $f(x) = -x^5 - 3x^4 + 42x^3 + 62x^2 - 297x - 315$ ARE:
$x = -7$
$x = -3$
$x = -1$
$x = 3$
$x = 5$

$(-7) \cdot (-3) \cdot (-1) \cdot 3 \cdot 5 = -315$

* PROBLEM 5.2

$f(x) = x^7 - 6x^6 + 8$



$f(0) = 0^7 - 6 \cdot 0^6 + 8 = 8$ +
$f(2) = 2^7 - 6 \cdot (2)^6 + 8 = -248$ −
$f(-2) = (-2)^7 - 6 \cdot (-2)^6 + 8 = -504$ −

-504	8	-248
$f(-2)$	$f(0)$	$f(2)$
↓	↓	↓
−	+	−

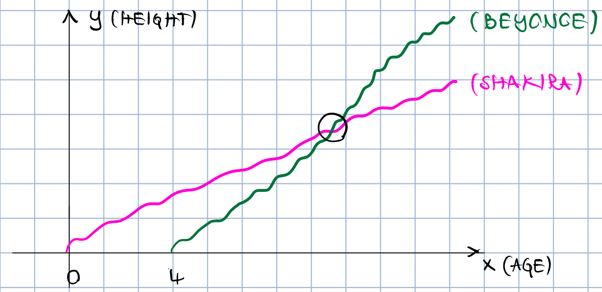
SINCE $f(-2)$ AND $f(0)$ HAVE DIFFERENT SIGNS WE CAN CONCLUDE THERE IS A ROOT IN THE INTERVAL $(-2, 0)$

SINCE $f(0)$ AND $f(2)$ HAVE DIFFERENT SIGNS WE CAN CONCLUDE THERE IS A ROOT IN THE INTERVAL $(0, 2)$

=> FROM THIS WE CAN CONCLUDE THAT THERE ARE TWO ROOTS AT THE INTERVAL $[-2, 2]$

* PROBLEM 5.3

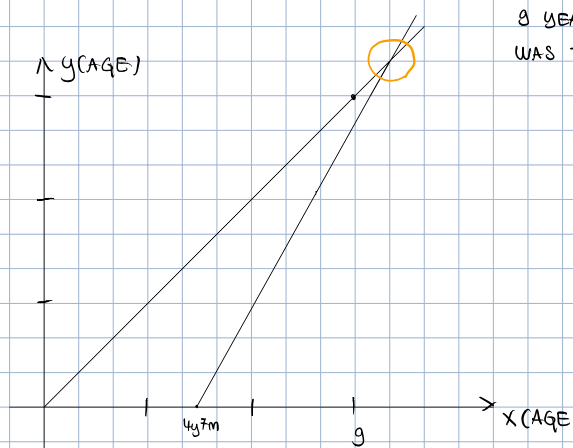
A) THERE WAS A MOMENT WHEN SHAKIRA WAS EXACTLY BEYONCE'S HEIGHT. EVENTHOUGH BEYONCE IS TALLER THAN SHAKIRA, SHAKIRA IS FOUR YEARS OLDER. THIS MEANS THAT IN ORDER TO BECOME TALLER, BEYONCE'S HEIGHT HAD TO CATCH UP WITH SHAKIRA'S HEIGHT OVER THE YEAR AND EVENTUALLY EXEED IT. => THIS MEANS THERE WAS A MOMENT WHEN SHAKIRA WAS EXACTLY BEYONCE'S HEIGHT. (GRAPH BELOW). GROWTH IS A CONTINOUS FUNCTION SO WE CAN ARGUE THIS.



B) THERE WAS A MOMENT WHEN SHAKIRA'S AGE WAS EXACTLY TWICE AS AGE OF BEYONCE.

BEYONCE – SEPTEMBER FOURTH 1981.
SHAKIRA – FEBRUARY SECOND 1977.

=) SHAKIRA IS 4 YEARS, 7 MONTHS, AND TWO DAYS OLDER THAN BEYONCE. THEREFORE WE CAN CONCLUDE THAT WHEN SHAKIRA WAS 9 YEARS, 2 MONTHS AND TWO DAYS OLD, SHE WAS TWICE AS AGE OF BEYONCE (GRAPH BELOW). AGE IS A CONTINOUS FUNCTION SO WE CAN ARGUE THIS.



S – SHAKIRA'S AGE
B – BEYONCE'S AGE
$S = 2B$

$S - 2B = 0$

$S - 2B = 0$

=) IN 2020 : $S = 43$ $B = 39$ $43 - 2 \cdot (39) = -35$

=) IN 1978 : $S = 1$ $B = 0$ $1 - 2 \cdot 0 = 1$

=) USING THE INTERMEDIATE VALUE THEOREM WE CAN CONCLUDE THIS IS TRUE

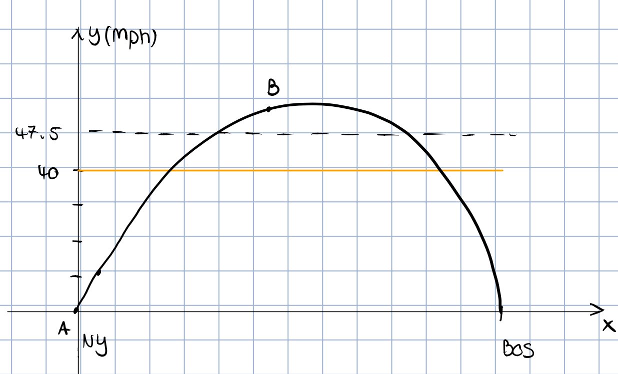
C) WE CANNOT ARGUE THAT THERE WAS A MOMENT WHEN SHAKIRA'S FORTUNE WAS EXACTLY HALF OF BEYONCE'S FORTUNE. WE CANNOT ARGUE THIS STATEMENT SIMPLY BECAUSE FORTUNE CANNOT BE REPRESENTED AS A CONTINOUS FUNCTION. THEREFORE WE CAN

D) BOSTON $\rightarrow$ NEW YORK

$S = 190$ MILES

$t = 4$ HOURS

$$v = \frac{S}{t} = \frac{190}{4} = 47.5$$



IN ORDER TO COMPLETE THIS TRIP ONE HAS TO TRAVEL AT THE AVERAGE SPEED OF 47.5 mph. ONE NEEDS TO PASS THROUGH THE POINT OF SPEED OF 40 mph AT LEAST TWO TIMES DURING THE TRIP. ONCE WHEN WE ARE SPEEDING UP TO 47.5 mph, AND ONCE WE ARE SLOWING DOWN FROM 47.5 mph.

★ PROBLEM 5.4

A) $5 - \sin(x) = x$

$-\sin(x) - x = -5$

$\sin(x) + x = 5$

$\sin(\pi) + \pi = 0 + 3.14 = 3.14$ (+)

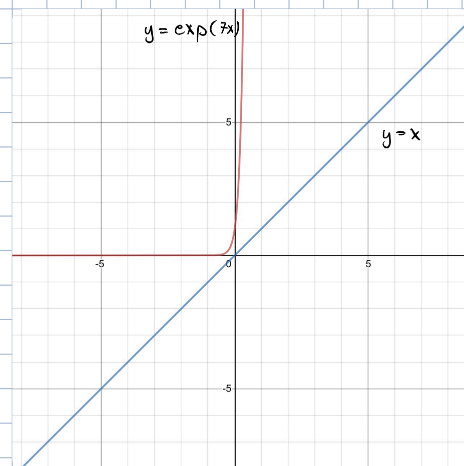
$\sin(2\pi) + 2\pi = 0 + 2\pi = 6.28$ (+)

$\Downarrow$

THESE TWO VALUES INDICATE THERE IS A SOLUTION FOR $5 - \sin(x) = x$

B) $\exp(7x) = x$

$\boxed{\Rightarrow \text{THERE IS NO SOLUTION}}$



C) $\sin(x) = x^4$ ⇒ FOR THIS PROBLEM WE CAN JUST PLUG IN THE ZERO AND SEE THERE IS A SOLUTION.

$$\boxed{\begin{array}{c}\sin(0) = 0^4 \\ 0 = 0\end{array}}$$

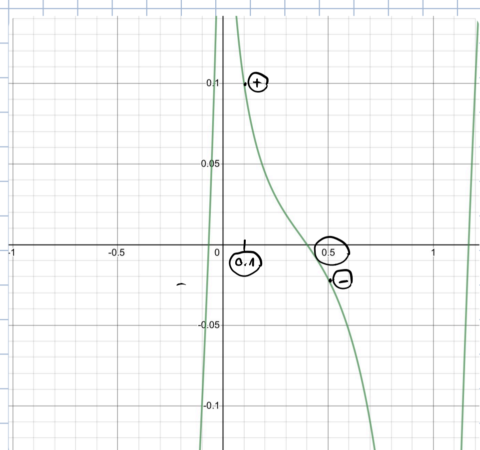
D) $f(x) = \dfrac{1}{\cos(x)}$

$f(0) = 1$

$f(\pi) = -1$

$\boxed{\Rightarrow \text{THIS IS NOT A CONTINOUS FUNCTION}}$

E)



$f(x) = x + \log|\log|x||$

$f(0.5) = 0.5 + \log(\log(0.5))$
$= 0.5 + \log(0.3)$
$= -1.023$ (−)

$f(0.1) = 0.1 + \log|\log|0.1||$
$= 0.1 + \log(1)$
$= 0.1$ (+)

USING THE INTERMEDIATE VALUE THEOREM WE CAN CONCLUDE THERE IS A ROOT IN THE INTERVAL $[0.1, 0.5]$

★ PROBLEM 5.5

A) $f(x) = -(x-1)(x-2)(x-3)(x+1)(x+2)(x+3)$

B) $f(x) = |x|$ $h = 1/10$

$$\frac{f(x+h) - f(x)}{h} = 0 \qquad \frac{f(x+1/10) - |x|}{1/10} = 0$$

$f(x+1/10) - |x| = 0$

$f(x+1/10) = |x|$

$$\frac{|x+h| - |x|}{1/10} = 0$$

$$x + \frac{1}{10} = -x$$

$2x + 1/10 = 0$

$|x+h| - |x| = 0$

$|x+h| = |x|$

$|x + 1/10| = |x|$

$x + 1/10 = -x$

$$2x = -\frac{1}{10}$$

$$\boxed{x = -\frac{1}{20}}$$

C) $f(x) = x^5$

$$\frac{f(x+h) - f(x)}{h} = 0$$

$f(x+h) = (x+h)^5$

$$\frac{(x+h)^5 - x^5}{h} = 0$$

$(x+h)^5 = x^5$

$x + h = x$

$\boxed{h > 0}$

THERE IS NO CRITICAL POINT BECAUSE THE FUNCTION IS MONOTON.