MATH 1A – PROBLEM SET 6. EVA GOLUZA

* PROBLEM 6.1

A) $f(x) = x^{11} - 3x^4 + 5x + 1$

$Df(x) = 11x^{10} - 12x^3 + 5$

C) $f(x) = -3x^3 + 17x^2 - 5x$

$Df(x) = -9x^2 + 3x^4 - 5$

B) $f(x) = -x^4 + 8\log(x)$

$f(x) = 8\log(x) - x^4$

$Df(x) = \frac{8}{x} - 4x^3$

D) $f(x) = \log(x+5)$

$Df(x) = \frac{1}{x+5}$

* PROBLEM 6.2

A) $f(x) = x^9 - 5$

$Sf(x) = \frac{x^{10}}{10} - 5x$

C) $f(x) = -3x^3 + 17x^2 - 5x$

$Sf(x) = -\frac{3}{4}x^4 + \frac{17}{3}x^3 - \frac{5}{2}x^2$

B) $f(x) = x^2 + 6x^7 - x$

$Sf(x) = \frac{x^3}{3} + \frac{6}{8}x^8 - \frac{x^2}{2}$

D) $f(x) = \exp(5x) + \sin(9x)$

$f(x) = e^{5x} + \sin(9x)$

$Sf(x) = \frac{e^{5x}}{5} - \frac{1}{9}\cos(9x)$

* PROBLEM 6.3

A) $f(x) = \exp(9 \cdot x + 3) + x^6$

$f(x) = e^{9x+3} + x^6$

$Df(x) = 9e^{9x+3} + 6x^5$

C) $f(x) = \log(1-x)$

$Df(x) = \frac{1}{1-x}$

B) $f(x) = 8\exp(-3 \cdot x) + 9x^6$

$f(x) = 8e^{-3x} + 9x^6$

$Df(x) = -24e^{-3x} + 54x^5$

D) $f(x) = \log(1-x^2)$

$f(x) = \log[(1+x)(1-x)]$

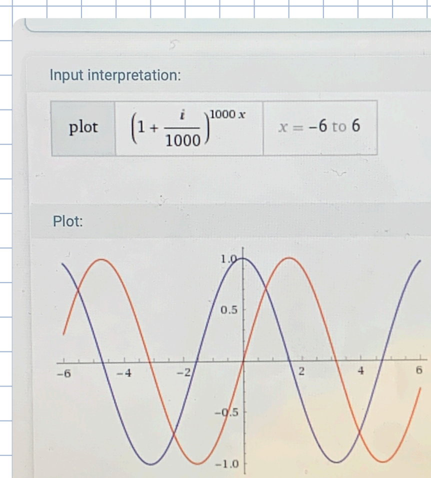
$f(x) = \log(1+x) + \log(1-x)$

$Df(x) = \frac{1}{1+x} + \frac{1}{1-x}$

* PROBLEM 6.4

A) $h = 1/100$

$$\left.\begin{array}{l} f(x) = \cos_h(x) \\ f(x) = \sin_h(x) \end{array}\right\} x \in [-2\pi, 2\pi]$$



B) THE LARGER h GETS, IT IS FURTHER AWAY FROM THE f(x)=sin FUNCTION.

IF h IS SMALLER IT GROWS FASTER

* PROBLEM 6.5

A) $f(x) = (1+h)^{x/h}$

$Df(x) = f(x)$

$$f(x) = \frac{f(x+h) - f(x)}{h} = \left[\frac{(1+h)^{\frac{x+h}{h}} - (1+h)^{x/h}}{h}\right]$$

$$= \frac{(1+h)^{\frac{x}{h}} \cdot (1+h)^{\frac{h}{h}} - (1+h)^{\frac{x}{h}}}{h}$$

$$= \frac{(1+h)^{\frac{x}{h}} \left[(1+h) - 1\right]}{h}$$

$$= \frac{(1+h)^{\frac{x}{h}} (1+h-1)}{h}$$

$$= \frac{(1+h)^{\frac{x}{h}} \cdot \cancel{h}}{\cancel{h}} = \boxed{(1+h)^{\frac{x}{h}}} \Rightarrow \boxed{\left[f(x+h) - f(x)\right]/h = f(x)}$$

B) $f(x) = x(x-h)(x-2h)(x-3h)(x-4h)$ $\quad D[x]^5 = 5[x]^4$

$$f(x) = \frac{f(x+h) - f(x)}{h}$$

$$f(x) = \frac{(x+h)(x+h-h)(x+h-2h)(x+h-3h)(x+h-4h) - x(x-h)(x-2h)(x-3h)(x-4h)}{h}$$

$$f(x) = \frac{\left[(x+h)(x)(x-h)(x-2h)(x-3h)\right] - \left[x(x-h)(x-2h)(x-3h)(x-4h)\right]}{h}$$

$$f(x) = \frac{\left[(x)(x-h)(x-2h)(x-3h)\right]\left[(x+h) - (x-4h)\right]}{h}$$

$$f(x) = \frac{\left[(x)(x-h)(x-2h)(x-3h)\right]\left[x+h-x+4h\right]}{h}$$

$$f(x) = \frac{\left[x(x-h)(x-2h)(x-3h)\right] \cdot 5\cancel{h}}{\cancel{h}}$$

$$\boxed{f(x) = x(x-h)(x-2h)(x-3h) \cdot 5 \Rightarrow \text{when } h=0 \quad f(x) = 5x^4}$$