MATH 1A – PROBLEM SET 7.

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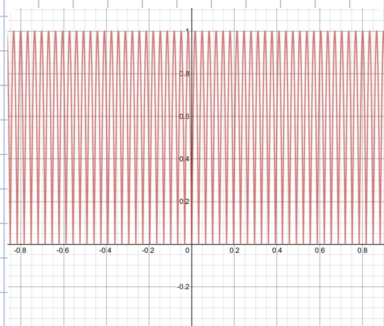
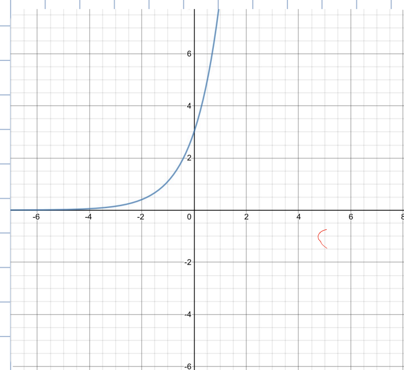
* PROBLEM 7.1

A) $|\sin(6x)|$



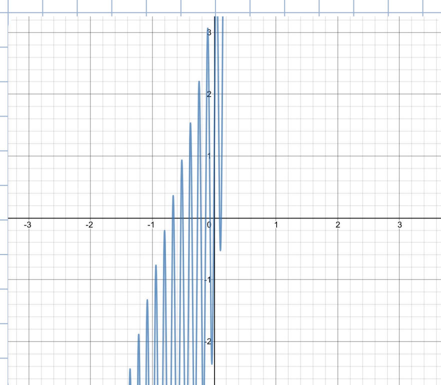

THE DERIVATIVE $f'(x)$ $\boxed{\text{DOES NOT EXSIST}}$ FOR EVERY $x$ IN $f(x)=|\sin(6x)|$. SINCE THIS IS A FUNCTION WITH ABSOLUTE VALUE, THE LIMITS ARE NOT APPROACHING THE SAME VALUE FROM POSITIVE AND NEGATIVE SIDE FOR EVERY $\frac{k\pi}{6}$

B) $|3\exp(x)|$



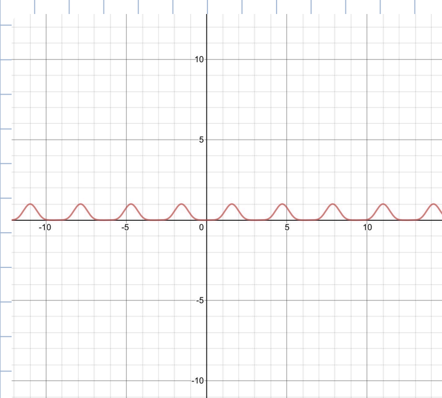

THE DERIVATIVE $f'(x)$ $\boxed{\text{EXISTS}}$ FOR EVERY $x$ OF $f(x)=|3\exp(x)|$. SINCE THIS IS AN EXPONENTIAL FUNCTION IT IS ALWAYS POSITIVE $\Rightarrow$ THE ABSOLUTE VALUE DOES NOT CHANGE ANYTHING.

C) $4x+\exp(7x)+3\sin(45x)$



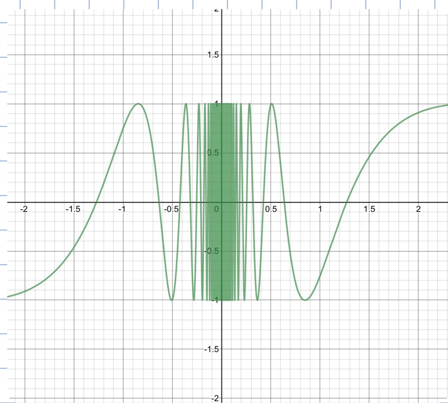

THIS IS THE SUM OF THREE CONTINOUS FUNCTIONS. THE DERIVATIVE $f'(x)$ $\boxed{\text{EXISTS}}$ FOR ALL $x$ IN $f(x)=4x+\exp(7x)+3\sin(45x)$

D) $\sin^6 x$



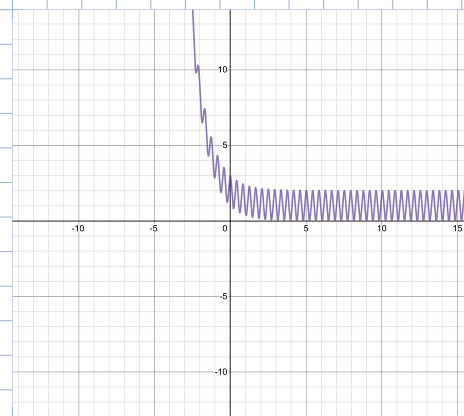

YES, THE DERIVATIVE $f'(x)$ $\boxed{\text{EXISTS}}$ FOR EVERY $x$ IN THE FUNCTION $f(x)=\sin^6 x$

E) $\sin(4/x)$




NO, THE DERIVATIVE $f'(x)$ DOES $\boxed{\text{NOT EXIST}}$ FOR EVERY X FOR $f(x) = \sin(4/x)$

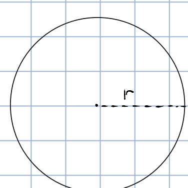
F) $\exp(-x) + |1 + \cos(15x)|$




YES, THE DERIVATIVE $f'(x)$ $\boxed{\text{EXISTS}}$ FOR EVERY X IN $f(x) = \exp(-x) + |1 + \cos(15x)|$

* PROBLEM 7.2

A)




$f(r) = \pi r^2$

$r = \frac{1}{10}$

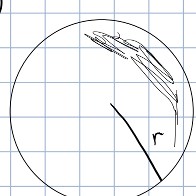
$f(r) = \pi r^2$

$f'(r) = 2\pi r$

$f'\left(\frac{1}{10}\right) = 2 \cdot \pi \cdot \frac{1}{10}$

$$\boxed{f'\left(\frac{1}{10}\right) = \frac{2\pi}{10} = \frac{\pi}{5}}$$

B)




$r = \frac{1}{10}$

$f(r) = 4\pi r^3/3$

$f(r) = 4\pi \frac{1}{3} r^3$

$f'(r) = 12\pi \cdot \frac{1}{3} \cdot r^2$

$f'(r) = 4\pi \cdot r^2$

$f'\left(\frac{1}{10}\right) = 4\pi \cdot \left(\frac{1}{10}\right)^2$

$f'\left(\frac{1}{10}\right) = 4\pi \cdot \frac{1}{100}$

$f'\left(\frac{1}{10}\right) = \frac{4\pi}{100}$

$$\boxed{f'\left(\frac{1}{10}\right) = \frac{\pi}{25}}$$

c) $r = \frac{1}{10}$

$f(r) = \pi^2 r^4/2$

$f(r) = \pi^2 \cdot \frac{1}{2} \cdot r^4$

$f'(r) = 4\pi^2 \cdot \frac{1}{2} \cdot r^3$

$f'(r) = 2\pi^2 \cdot r^3$

$f'(\frac{1}{10}) = 2\pi^2 \cdot (\frac{1}{10})^3$

$f'(\frac{1}{10}) = 2\pi^2 \cdot \frac{1}{1000}$

$f'(\frac{1}{10}) = \frac{2\pi^2}{1000}$

$\boxed{f'(\frac{1}{10}) = \frac{\pi^2}{500}}$

* PROBLEM 7.3

A) $f(x) = 7\exp(4x) + \sin(11x) + x + x^4 + x^6 + x^{10}$

$f'(x) = 28e^{4x} + 11\cos(11x) + 1 + 4x^3 + 6x^5 + 10x^9$

$f'(0) = 28 \cdot 1 + 11 \cdot 1 + 1 + 0 + 0 + 0$

$f'(0) = 28 + 11 + 1$

$\boxed{f'(0) = 40}$

B) $f(x) = \frac{(x^5-1)}{x-1} + \cos(2x)$

$f(x) = \frac{\cancel{(x-1)}(x^4+x^3+x^2+x+1)}{\cancel{(x-1)}} + \cos(2x)$

$f(x) = (x^4+x^3+x^2+x+1) + \cos(2x)$

$f'(x) = 4x^3 + 3x^2 + x + 1 - 2\sin(2x)$

$\boxed{f'(0) = 1}$

C) $f(x) = \frac{1 + 4x + 6x^2 + 4x^3 + x^4}{x^2 + 2x + 1}$

=> the coefficients match the 5th row of Pascal's triangle

=> $f(x) = \frac{(x+1)^4}{(x+1)^2}$

$f(x) = (x+1)^2$

$\boxed{f'(0) = 2}$

$f(x) = x^2 + 2x + 1$

$f'(x) = 2x + 2$

$f'(0) = 2$

## * PROBLEM 7.4

$$\lim_{x\to 0} \frac{\sqrt{x+h}-\sqrt{x}}{h} \cdot \frac{\sqrt{x+h}+\sqrt{x}}{\sqrt{x+h}+\sqrt{x}}$$

$$\lim_{x\to 0} \frac{(\sqrt{x+h})^2-(\sqrt{x})^2}{h(\sqrt{x+h}+\sqrt{x})}$$

$$\lim_{x\to 0} \frac{x+h-x}{h(\sqrt{x+h}+\sqrt{x})}$$

$$\lim_{x\to 0} \frac{h}{h(\sqrt{x+h}+\sqrt{x})}$$

$$\lim_{x\to 0} \frac{1}{(\sqrt{x+h}+\sqrt{x})}$$

$$\lim_{x\to 0} \frac{1}{\sqrt{x+h}+\sqrt{x}}$$

$$\lim_{x\to 0} \frac{1}{2\sqrt{x}}$$

$$\boxed{\lim_{x\to 0} \frac{\sqrt{x}}{2x}}$$

## * PROBLEM 7.5

$h(t)=\sqrt{t^2+20^2}$

$t=0$

$$\lim_{x\to 0} \frac{f(x+h)-f(x)}{h}$$

$$\lim_{x\to 0} \frac{\sqrt{(t+h)^2+20^2}}{h} - \frac{\sqrt{t^2+20^2}}{h}$$

$$\lim_{x\to 0} \frac{\sqrt{(t+h)^2+400}-\sqrt{t^2+400}}{h}$$

$$\lim_{x\to 0} \frac{\sqrt{(t+h)^2+400}-\sqrt{t^2+400}}{h} \cdot \frac{\sqrt{(t+h)^2+400}+\sqrt{t^2+400}}{\sqrt{(t+h)^2+400}+\sqrt{t^2+400}}$$

$$\lim_{x\to 0} \frac{(t+h)^2+400-(t^2+400)}{h\cdot(\sqrt{(t+h)^2+400}+\sqrt{t^2+400})}$$

$$\lim_{x\to 0} \frac{\cancel{t^2}+2th+h^2+\cancel{400}-\cancel{t^2}-\cancel{400}}{h(\sqrt{(t+h)^2+400}+\sqrt{t^2+400})}$$

$$\lim_{x\to 0} \frac{\cancel{h}(2t+h)}{\cancel{h}(\sqrt{(t+h)^2+400}+\sqrt{(t^2+400)})}$$

$$\lim_{x\to 0} \frac{2t+h}{(\sqrt{(t+h)^2+400}+\sqrt{(t^2+400)})}$$

$$\frac{2t+0}{(\sqrt{(t+0)^2+400}+\sqrt{(t^2+400)})}$$

$$\frac{2t}{\sqrt{t^2+400}+\sqrt{t^2+400}}$$

$$\frac{\cancel{2}t}{\cancel{2}\left(\sqrt{t^2+400}\right)}$$

$$\frac{t}{\sqrt{t^2+400}}$$

$$\boxed{\frac{0}{20} = 0}$$