MATH 1A – PROBLEM SET 9

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★ PROBLEM 9.1

A) $f(x) = \sin(3x) + \sin(120x)\tan(121x)$

$\cos(0)=1$
$\sin(0)=0$

$$f'(x) = \frac{d}{dx}(\sin(3x)) + \frac{d}{dx}(\sin(120x)\tan(121x))$$

$$f'(x) = \frac{d}{dx}(\sin(3x)) + \left[\frac{d}{dx}(\sin(120x))\cdot\tan(121x) + \sin(120x)\cdot\frac{d}{dx}(\tan(121x))\right.$$

$$f'(x) = 3\cos(3x) + \left[120\cos(120x)\cdot\tan(121x) + \sin(120x)\cdot\frac{1}{121\cos^2(121x)}\right.$$

$$f'(x) = 3\cos(3x) + \left[120\cos(120x)\cdot\tan(121x) + \frac{\sin(120x)}{121\cos^2(121x)}\right]$$

$$f'(x) = 3\cos(3x) + \left[120\cos(120x)\cdot\frac{\sin(121x)}{\cos(121x)} + \frac{\sin(120x)}{121\cos^2(121x)}\right]$$

$$f'(x) = \frac{3\cos^2(121x)\cos(3x) + 60\sin(242x)\cos(120x) + 121\sin(120x)}{\cos^2(121x)}$$

$$f'(0) = \frac{3\cos^2(0)\cos(0) + 60\sin(0)\cos(0) + 121\sin(0)}{\cos^2(0)}$$

$$f'(0) = \frac{3\cdot1\cdot1 + 60\cdot0\cdot1 + 121\cdot0}{1}$$

$$f'(0) = \frac{3+0+0}{1} = \frac{3}{1}$$

$$\boxed{f'(0) = 3}$$

B) $f(x) = \dfrac{\cos^4 x}{(1+x)^5}$

$$f'(x) = \frac{d}{dx}\left(\frac{\cos^4 x}{(1+x)^5}\right)$$

$$f'(x) = \frac{\frac{d}{dx}(\cos^4 x)\cdot(1+x)^5 - \cos^4 x\cdot\frac{d}{dx}(1+x)^5}{((1+x)^5)^2}$$

$$f'(x) = \frac{(-4\sin(x)\cos^3 x\cdot(1+x)^5)\cdot(\cos^4 x\cdot 5(1+x)^4}{((1+x)^5)^2}$$

$$f'(x) = -\frac{4\cos^3(x)\sin(x)\cdot(1+x) + 5\cos^4(x)}{(1+x)^6}$$

$$f(x) = -\frac{\cos^3(x)(4(1+x)\sin(x) + 5\cos(x))}{(1+x)^6}$$

$$f'(0) = -\frac{\cos^3(0)(4\sin(0) + 5\cos(x))}{(1)^6}$$

$$\boxed{f'(0) = -5}$$

C) $f(x) = e^x \sin(x)\cos(x)$

$f(x) = \frac{1}{2} e^x \sin(2x)$

$f'(x) = \frac{1}{2} \cdot \frac{d}{dx}(e^x \cdot \sin(2x)$

$f'(x) = \frac{1}{2} \cdot \left(\frac{d}{dx}(e^x) \cdot \sin(2x) + e^x \cdot \frac{d}{dx}(\sin 2x)\right)$

$f'(x) = \frac{1}{2} \cdot \left(e^x \cdot \sin(2x) + e^x \cdot 2\cos(2x)\right)$

$f'(0) = \frac{1}{2} \cdot \left(e^0 \cdot \sin(0) + e^0 \cdot 2\cos(0)\right)$

$f'(0) = \frac{1}{2} \cdot (1 \cdot 0 + 1 \cdot 2)$

$f'(0) = \frac{1}{2} \cdot 2$

$\boxed{f'(0) = 1}$

D) $f(x) = \frac{3}{\cos(x)} + \frac{1}{\sqrt{x+1}}$

$f'(x) = \frac{d}{dx}\left(\frac{3}{\cos x}\right) + \frac{d}{dx}\left(\frac{1}{\sqrt{x+1}}\right)$

$f'(x) = \frac{\frac{d}{dx}(3) \cdot \cos x - 3 \cdot \frac{d}{dx}(\cos x)}{\cos^2 x} + \frac{\frac{d}{dx}(1) \cdot \sqrt{x+1} - 1 \cdot \frac{d}{dx}(\sqrt{x+1})}{(\sqrt{x+1})^2}$

$f'(x) = -3 \cdot \frac{(-\sin x)}{\cos^2(x)} - \frac{\frac{1}{2\sqrt{x+1}}}{(\sqrt{x+1})^2}$

$f'(x) = \frac{3\sin(x)}{\cos^2(x)} - \frac{1}{(2\sqrt{x+1})(x+1)}$

$f'(0) = \frac{3\sin(0)}{\cos^2(0)} - \frac{1}{(2\sqrt{0+1})(0+1)}$

$f'(0) = \frac{3 \cdot 0}{1} - \frac{1}{2\sqrt{1} \cdot 1}$

$f'(0) = 0 - \frac{1}{2}$

$\boxed{f'(0) = -\frac{1}{2}}$

E) $f(x) = 6xe^{5x} + 8\tan(x)$

$$f'(x) = \frac{d}{dx}\left(6xe^{5x} + 8\tan(x)\right)$$

$$f'(x) = \frac{d}{dx}(6xe^{5x}) + \frac{d}{dx}(8\tan(x))$$

$$f'(x) = \left[\frac{d}{dx}(6x)\cdot e^{5x} + 6x\cdot\frac{d}{dx}(e^{5x})\right] + \frac{d}{dx}(8\tan(x))$$

$$f'(x) = 6e^{5x} + 30xe^{5x} + \frac{8}{\cos^2(x)}$$

$$f'(0) = 6e^{0} + 30\cdot 0\cdot e^{0} + \frac{8}{\cos^2(0)}$$

$$f'(0) = 6\cdot 1 + 30\cdot 0\cdot 1 + 8$$

$$f'(0) = 6 + 0 + 8$$

$$f'(0) = 14$$

$$\boxed{f'(0) = 14}$$

* PROBLEM 9.3

$f(x) = \mathrm{sinc}(x)$

$$f(x) = \frac{\sin(x)}{x}$$

$$f'(x) = \frac{\frac{d}{dx}(\sin(x))\cdot x - \sin(x)\cdot\frac{d}{dx}(x)}{x^2}$$

$$f'(x) = \frac{\cos(x)\cdot x - \sin(x)}{x^2}$$

$f'(x) = g(x)$

$$g(x) = \frac{\cos(x)\cdot x - \sin(x)}{x^2}$$

$$g'(x) = \frac{d}{dx}\left(\frac{\cos(x)\cdot x - \sin(x)}{x^2}\right)$$

$$g'(x) = \frac{d}{dx}\left(\frac{\cos(x)x}{x^2} - \frac{\sin(x)}{x^2}\right)$$

$$g'(x) = \frac{d}{dx}\left(\frac{\cos(x)}{x}\right) - \frac{d}{dx}\left(\frac{\sin(x)}{x^2}\right)$$

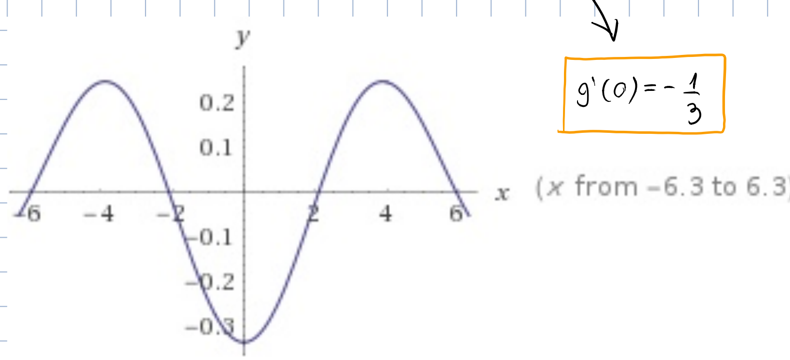
$$g'(x) = \frac{\frac{d}{dx}(\cos(x))\cdot x - \cos(x)\cdot\frac{d}{dx}(x)}{x^2} - \frac{\frac{d}{dx}(\sin(x))\cdot x^2 - \sin(x)\cdot\frac{d}{dx}(x^2)}{x^4}$$

$$g'(x) = \frac{-\sin\cdot x - \cos(x)}{x^2} - \frac{\cos(x)\cdot x^2 - \sin(x)\cdot 2x}{x^4}$$

$$g'(x) = \frac{-x^2\cdot\sin(x) - 2x\cdot\cos(x) + 2\sin(x)}{x^3}$$

$$g'(x) = -\frac{(x^2-2)\sin(x) + 2x\cdot\cos(x)}{x^3}$$

$$\boxed{g'(0) = -\frac{1}{3}}$$



★ PROBLEM 9.4

A)
$$f(x)=\frac{\sin(x)}{1+\cos(x)+\frac{x^4}{\sin(x)}}$$

$$f'(x)=\frac{d}{dx}\left(\frac{\frac{\sin(x)}{1}}{\frac{\sin(x)+\sin(x)\cos(x)+x^4}{\sin(x)}}\right)$$

$$f'(x)=\frac{d}{dx}\left(\frac{\sin^2(x)}{\sin(x)+\sin(x)\cos(x)+x^4}\right)$$

$$f'(x)=\frac{\frac{d}{dx}(\sin^2 x)\cdot(\sin(x)+\sin(x)\cos(x)+x^4)-\sin^2(x)\cdot\frac{d}{dx}(\sin(x)+\sin(x)\cos(x)+x^4)}{(\sin(x)+\sin(x)\cos(x)+x^4)^2}$$

$$f'(x)=\frac{2\sin(x)\cos(x)\cdot(\sin(x)+\sin(x)\cos(x)+x^4)-\sin^2(x)\cdot(\cos(x)+\cos(x)\cos(x)+\sin(x)\cdot(-\sin(x))+4x^3)}{(\sin(x)+\sin(x)\cos(x)+x^4)^2}$$

$$f'(x)=\frac{\sin(2x)\sin(x)+\sin(2x)\sin(x)\cos(x)+\sin(2x)\cdot x^4-\sin^2(x)\cos(x)-\sin^2(x)\cos(2x)-4x^3\cdot\sin^2(x)}{(\sin(x)+\sin(x)\cos(x)+x^4)^2}$$

$$f'(x)=\frac{2x^4\cot(x)-4x^3+\sin^2(x)+\cos^2(x)+\cos(x)}{(x^4\csc(x)+\cos(x)+1)^2}$$

$$f'(0)=\frac{2\cdot 0\cdot\cot(0)-4\cdot 0+0+1+1}{(0\cdot\csc(x)+1+1)^2}$$

$$f'(0)=\frac{2}{2^2}=\frac{2}{4}$$

$$\boxed{f'(0)=\frac{1}{2}}$$

B)
$$f(x)=\frac{1}{\sin(x)+\frac{1}{\sin(x)+1/\sin(x)}}$$

$$f'(x)=\frac{d}{dx}\left(\frac{1}{\sin(x)+\frac{1}{\sin(x)+1/\sin(x)}}\right)$$

$$f'(x)=\frac{d}{dx}\left(\frac{1}{\sin(x)+\frac{1}{\frac{\sin^2(x)+1}{\sin(x)}}}\right)$$

$$f'(x) = \frac{d}{dx}\left(\frac{1}{\sin(x) + \frac{\sin(x)}{\sin^2(x)+1}}\right)$$

$$f'(x) = \frac{d}{dx}\left(\frac{1}{\frac{(\sin^2 x + 1)\cdot\sin(x) + \sin(x)}{\sin^2(x)+1}}\right)$$

$$f'(x) = \frac{d}{dx}\left(\frac{1}{\frac{\sin^3(x) + \sin(x) + \sin(x)}{\sin^2(x)+1}}\right)$$

$$f'(x) = \frac{d}{dx}\left(\frac{1}{\frac{\sin^3(x)+2\sin(x)}{\sin^2(x)+1}}\right)$$

$$f'(x) = \frac{d}{dx}\left(\frac{\sin^2(x)+1}{\sin^3(x)+2\sin(x)}\right)$$

$$f'(x) = \frac{\frac{d}{dx}(\sin^2(x)+1)\cdot(\sin^3(x)+2\sin(x)) - (\sin^2(x)+1)\cdot\frac{d}{dx}(\sin^3(x)+2\sin(x))}{(\sin^3(x)+2\sin(x))^2}$$

$$f'(x) = \frac{[2\sin(x)\cos(x)\cdot(\sin^3(x)+2\sin(x))] - [(\sin^2(x)+1)\cdot(3\sin^2(x)\cos(x)+2\cos(x))]}{(\sin^3(x)+2\sin(x))^2}$$

$$f'(x) = \frac{\sin(2x)\sin^3(x) + 2\sin(2x)\sin(x) - 3\sin^4(x)\cos(x) - 5\sin^2(x)\cos(x) - 2\cos(x)}{(\sin^3(x)+2\sin(x))^2}$$

$$f'\left(\frac{\pi}{2}\right) = \frac{\csc^2(x)(\cos(x) + \cot(x)\csc(x)(2\csc^2(x)+1))}{(2\csc^2(x)+1)^2}$$

$$f'\left(\frac{\pi}{2}\right) = \frac{1\cdot 0 + 0\cdot 1\cdot 0}{9}$$

$\sin(\pi) = 0$

$\cos(\pi) = -1$

$\sin\left(\frac{\pi}{2}\right) = 1$

$\cos\left(\frac{\pi}{2}\right) = 0$

$\csc\left(\frac{\pi}{2}\right) = \frac{1}{1}$

$$f'\left(\frac{\pi}{2}\right) = 0$$

$$\boxed{f'\left(\frac{\pi}{2}\right) = 0}$$

* PROBLEM 9.2

A) $f(x) = g(x)h(x)k(x)\ell(x)$

$f'(x) = g'hk\ell + gh'k\ell + ghk'\ell + ghk\ell'$

$f = (gh)(k\ell)$

$f' = (g'h + gh')(k\ell) + (gh)(h'\ell + h\ell')$

$f' = g'hk\ell + gh'k\ell + ghk'\ell + ghk\ell'$

B) $f(x) = g(x)^4$

$f'(x) = 4g^3(x)g'(x)$

$f(x) = g(x)g(x)g(x)g(x)$

$f'(x) = g'(x)\cdot g(x)\cdot g(x)\cdot g(x) + g(x)\cdot g'(x)\cdot g(x)\cdot g(x) + g'(x)g(x)g(x)g(x) + g'(x)g(x)g(x)g(x)$

$f'(x) = 4g^3(x)g'(x)$

* PROBLEM 9.5

A) $g(x) = \sqrt{x}$ $\quad f(x) = x^{1/4}$

$g(x) = f(x)\cdot f(x)$

$g'(x) = f'(x)f(x) + f'(x)f(x)$

$= 2f'(x)f(x)$

$\sqrt{x} = x^{\frac{1}{4}}\cdot x^{\frac{1}{4}}$

$g'(x) = \frac{d}{dx}(x)^{-\frac{1}{4}}\cdot x^{\frac{1}{4}} + x^{\frac{1}{4}}\cdot\frac{d}{dx}\left(x^{-\frac{1}{4}}\right)$

$\frac{1}{2}x^{-1/2} = 2x^{1/4}\cdot\frac{d}{dx}x^{1/4}$

$$\boxed{\frac{1}{4}x^{-\frac{3}{4}} = \frac{d}{dx}(x)^{\frac{1}{4}}}$$

B) $\frac{d}{dx}f = 1 = 4g^3(x)g'(x)$

$4\cdot x^{3/4}\cdot g'(x)$

$4x^{-3/4} = g'(x)$

C) $f(x) = x^{\frac{1}{4}}$

$g(x) = \sqrt{x} \Rightarrow g(x) = f(x)f(x) = x^{\frac{1}{4}}$

$g'(x) = \frac{1}{4}x^{-3/4}\cdot x^{1/4} + x^{1/4}\cdot x^{-3/4} = \frac{1}{2}\cdot x^{-\frac{1}{2}}$

$g'(x) = f'(x)f(x) + f(x)f'(x)$

$g'(x) = 2f'(x)f(x) = \frac{1}{2}x^{-\frac{1}{2}}$

$f'(x)\cdot x^{\frac{1}{4}} = \frac{1}{4}x^{-\frac{1}{2}}$

$f'(x) = \frac{1}{4x^{1/2}\cdot x^{\frac{1}{4}}}$

$f'(x) = \frac{1}{4x^{3/4}}$

$$\boxed{\frac{d}{dx}\left(x^{\frac{1}{4}}\right) = \frac{1}{4}x^{-3/4}}$$

THIS IS WHAT THEY TOLD ME AT THE MQC

C) $x = g(x)^4$

$$\boxed{1 = 4\cdot g(x)^3\cdot g'(x)}$$

THIS IS WHAT OLIVER TOLD ME TO DO DURING OFFICE HOURS, SO I AM KINDA CONFUSED

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