MATH 1A - PROBLEM SET 12

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* PROBLEM 1.

$f(x) = 3x^4 - 8x^3 - 6x^2 + 24x$ $\qquad [-2, 3]$

$f'(x) = \frac{d}{dx}(3x^4 - 8x^3 - 6x^2 + 24x)$

$f'(x) = 12x^3 - 24x^2 - 12x + 24$

$f'(x_0) = 0$

$12x^3 - 24x^2 - 12x + 24 = 0$

$x_1 = -1$

$x_2 = 2$

$x_3 = 1$

$f''(x) = \frac{d}{dx}(12x^3 - 24x^2 - 12x + 24)$

$f''(x) = 36x^2 - 48x - 12$

$f''(-1) = 36 + 48 - 12 = 72$ $\qquad 72 > 0 \Rightarrow x = -1$ IS A LOCAL MINIMUM

$f''(2) = 36 \cdot 4 - 96 - 12 = 36$ $\qquad 36 > 0 \Rightarrow x = 2$ IS A LOCAL MINIMUM

$f''(1) = 36 - 48 - 12 = -24$ $\qquad -24 < 0 \Rightarrow x = 1$ IS A LOCAL MAXIMUM

CRITICAL POINTS

$f(x) = 3x^4 - 8x^3 - 6x^2 + 24x$

$f(-1) = 3 \cdot 1 + 8 - 6 - 24$

$f(-1) = 3 + 8 - 6 - 24$

$f(-1) = -19$

$f(2) = 3 \cdot 16 - 8 \cdot 8 - 6 \cdot 4 + 24 \cdot 2$

$f(2) = 48 - 64 - 24 + 48$

$f(2) = 8$

$\Rightarrow$ THE GLOBAL MINIMUM IS AT $x = -1$

THE GLOBAL MAXIMUM IS AT $x = 3$

$f(-2) = 3 \cdot 16 + 8 \cdot 8 - 6 \cdot 4 - 48$

$f(-2) = 48 + 64 - 24 - 48$

$f(-2) = 40$

$f(3) = 3 \cdot 81 - 8 \cdot 27 - 6 \cdot 9 + 24 \cdot 3$

$f(3) = 243 - 216 - 54 + 72$

$f(3) = 45$

* PROBLEM 2.

$f(x) = 2x^3 - 3x^2 - 36x$ $\qquad [-4, 4]$

$f'(x) = 6x^2 - 6x - 36$

$f'(x_0) = 0$ $\qquad f''(x) = 6x - 6$

$6x^2 - 6x - 36 = 0$

$x_1 = 3$ $\qquad f''(3) = 6 \cdot 3 - 6 = 18 - 6 = 12$ $\qquad 12 > 0 \Rightarrow x = 3$ IS A LOCAL MINIMUM

$x_2 = -2$ $\qquad f''(-2) = -12 - 6 = -18$ $\qquad -18 < 0 \Rightarrow x = -2$ IS A LOCAL MAXIMUM

$f(x) = 2x^3 - 3x^2 - 36x$

$f(3) = 2 \cdot 27 - 3 \cdot 9 - 36 \cdot 3$

$f(3) = -81$

$f(-2) = 2 \cdot (-8) - 3 \cdot 4 + 36 \cdot 2$

$f(-2) = 44$

$\Rightarrow$ GLOBAL MAXIMUM IS AT $x = -2$

GLOBAL MINIMUM IS AT $x = 3$

$f(-4) = -2 \cdot 64 - 3 \cdot 16 + 36 \cdot 4$

$f(-4) = -32$

$f(4) = 2 \cdot 64 - 3 \cdot 16 - 36 \cdot 4$

$f(4) = -64$

* PROBLEM 3

A(r) = surface area of a candy radius r
V(r) = volume of a candy radius r
effectiveness = A(r) - V(r)

$f(r) = A(r) - V(r) \qquad r \geq 0$

$A(r) = 4\pi r^2$

$V(r) = \frac{4}{3}\pi r^3$

$\Rightarrow \quad f(r) = 4\pi r^2 - \frac{4}{3}\pi r^3$

$f'(r) = \frac{d}{dx}\left(4\pi r^2 - \frac{4}{3}\pi r^3\right)$

$f'(r) = 8\pi r - 4\pi r^2$

$f'(r_0) = 0$

$8\pi r - 4\pi r^2 = 0$

$r_1 = 0$

$r_2 = 2$

$f''(r) = 8\pi - 8\pi r$

$f''(2) = \ominus \Rightarrow r = 2$ IS A GLOBAL MAXIMUM

$f''(0) = \oplus \Rightarrow r = 0$ IS A GLOBAL MINIMUM

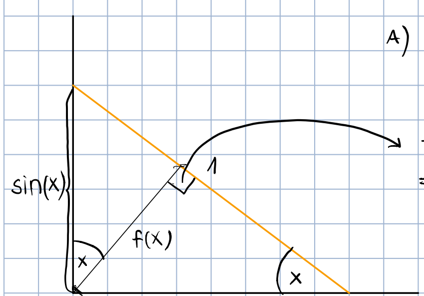
$f(0) = 0$

$f(2) = 16.76 \quad \Rightarrow \quad \boxed{\text{GLOBAL MAXIMUM IS } r = 2}$

$f(2.1) = 16.62$

* PROBLEM 4

A) $f(x) = \sin(x)\cos(x)$



THIS ANGLE IS EQUAL TO 90°
$\Rightarrow$ IF WE WANT TO MINIMIZE THE DISTANCE FROM THE LATER TO THE CORNER THE ANGLE SHOULD BE EQUAL TO 90 DEGREES

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THESE TWO TRIANGLES ARE SIMMILAR

$\cos(x) = \frac{f(x)}{\sin(x)} \Rightarrow \boxed{f(x) = \sin(x)\cos(x)}$

BOUNDRIES: $\frac{\pi}{2}, 0$

$f\left(\frac{\pi}{2}\right) = 0$

$f(0) = 0$

B) $f(x) = \sin(x)\cos(x)$

$f'(x) = \frac{d}{dx}(\sin(x)\cos(x))$

$f'(x) = \frac{d}{dx}(\sin(x)) \cdot \cos(x) + \sin(x) \cdot \frac{d}{dx}(\cos(x))$

$f'(x) = \cos(x) \cdot \cos(x) + \sin(x) \cdot (-\sin(x))$

$f'(x) = \cos(x) \cdot \cos(x) - \sin(x) \cdot \sin(x)$

$f'(x) = \cos^2(x) - \sin^2(x)$

$f'(x_0) = 0$

$\cos^2(x) - \sin^2(x) = 0$

$\boxed{x = \frac{\pi}{4}}$

BOUNDRIES $\frac{\pi}{2}, 0$

$f\left(\frac{\pi}{2}\right) = 0$

$f(0) = 0$

★ PROBLEM 5.

$S(x) = -x\log(x) \qquad 0 < x \leq 1$

$S'(x) = \frac{d}{dx}(-x\log(x))$

$S'(x) = \frac{d}{dx}(-x)\cdot\log(x) - x\cdot\frac{d}{dx}(\log(x))$

$S'(x) = -\log(x) - 1 \qquad f(0) = 0$

$S'(x_0) = 0 \qquad f(1) = 0$

$-\log(x) - 1 = 0$

$-\log(x) = 1$

$\log(x) = -1$

$x = e^{-1}$

$$\boxed{x = \frac{1}{e}}$$