MATH 1A - PROBLEM SET (14.)

$$T(x) = x - \frac{f(x)}{f'(x)}$$

* PROBLEM 14.1

A) $f(x) = (x-2)^2$

$f(x) = x^2 - 4x + 4$

$f'(x) = 2x - 4$

$$T(x) = x - \frac{(x-2)^2}{2x-4} = x - \frac{(x-2)^2}{2(x-2)} = x - \frac{(x-2)}{2} = \frac{2x-(x-2)}{2} = \frac{2x-x+2}{2} = \frac{x+2}{2}$$

$$\boxed{T(x) = \frac{x+2}{2}}$$

B) $f(x) = e^{5x}$

$f'(x) = 5e^{5x}$

$$T(x) = x - \frac{e^{5x}}{5e^{5x}} = x - \frac{1}{5}$$

$$\boxed{T(x) = x - \frac{1}{5}}$$

C) $f(x) = 2e^{-x^2}$

$f'(x) = -4xe^{-x^2}$

$$T(x) = x - \frac{2e^{-x^2}}{-4xe^{-x^2}} = x + \frac{2e^{-x^2}}{4xe^{-x^2}} = x + \frac{2}{4x} = x + \frac{1}{2x} = \frac{2x^2+1}{2x}$$

$$\boxed{T(x) = \frac{2x^2+1}{2x}}$$

D) $f(x) = \cot(x)$

$f'(x) = -\csc^2(x)$

$f'(x) = -\frac{1}{\sin^2(x)}$

$$T(x) = x - \frac{\cot(x)}{(-\csc^2(x))} = x + \frac{\cot(x)}{\csc^2(x)} = x + \frac{\frac{\cos(x)}{\sin(x)}}{\frac{1}{\sin^2(x)}} = x + \sin(x)\cos(x)$$

$$\boxed{T(x) = x + \sin(x)\cos(x)}$$

* PROBLEM 14.2

$f(x) = \cos(x) - x$

$f'(x) = -\sin(x) - 1$

$$T(x) = x - \frac{\cos(x)-x}{(-\sin(x)-1)} = x + \frac{\cos(x)-x}{\sin(x)-1}$$

$$T(1) = 1 + \frac{\cos(1)-1}{\sin(1)-1} = 0.7503$$

$$T(0.7503) = 1 + \frac{\cos(0.7503-1)}{\sin(0.7503-1)}$$

$$\boxed{T(0.753) = 0.7391}$$

* PROBLEM 14.4

$f(x) = \frac{1}{x}$

$f'(x) = -\frac{1}{x^2}$

$$T(x) = x - \frac{\frac{1}{x}}{\left(-\frac{1}{x^2}\right)} = x + \frac{\frac{1}{x}}{\frac{1}{x^2}} = x + x = 2x$$

$T(1) = 2$
$T(2) = 4$
$T(4) = 8$
$T(8) = 16$

THE METHOD DOES NOT CONVERGE IN THIS CASE.
THE VALUE DOUBLES WITH EVERY STEP MADE

* PROBLEM 14.3

$\sqrt{102} = x$

$f(x) = x^2 - 102 = 0$

START ($x = 10$)

$f(x) = x^2 - 102$

$f'(x) = 2x$

$$T(x) = x - \frac{f(x)}{f'(x)}$$

$$T(10) = 10 - \frac{100 - 102}{20} = 10 - \frac{(-2)}{20} = 10 + \frac{1}{10} = \frac{101}{10} = 10.1$$

$x_1 = 10.1$

$$T(10.1) = 10.1 - \frac{(10.1)^2 - 102}{20.2}$$

$T(10.1) =$ 10.0995

* PROBLEM 14.5

- $f(x) = x^{10} + x^4 - 20x$, $(1/2)$
- $g(x) = f'(x) - 1018$

$g(x) = 10x^9 + 4x^3 - 1038 = 0$

$f(1) = 1^{10} + 1^4 - 20 = 1 + 1 - 20 = -18$

$f(2) = 2^{10} + 2^4 - 40 = 1000$

$f(x) = x^{10} + x^4 - 20x$

$f'(x) = 10x^9 + 4x^3 - 20$

$g(x) = 10x^9 + 4x^3 - 20$

$g(1018) =$

ANY INTERVAL (a, b) CONTAINS A POINT X SUCH THAT

$$f'(x) = \frac{f(b) - f(a)}{b - a}$$

$$f'(x) = \frac{1000 - (-18)}{2 - 1}$$

$f'(x) = 1000 + 18$

$f'(x) = 1018$

$$T(x) = x - \frac{g(x)}{g'(x)}$$

$$T(1.5) = 1.5 - \frac{10x^9 + 4x^3 - 1038}{90x^8 + 12x^2}$$

$$T(1.5) = 1.7742$$