MATH 1A - PROBLEM SET 17.

$$\boxed{\int_a^b f(x)\,dx = F(b) - F(a) \qquad F'(x) = f(x)}$$

* PROBLEM 17.1

A) $\int_1^2 x^8\,dx = F(2) - F(1)$

$F'(x) = f(x)$

$F'(x) = x^8$

$F(x) = \frac{x^9}{9}$

$f'(x^n) \Rightarrow f(x) = \frac{x^{n+1}}{n+1}$

$= \frac{2^9}{9} - \frac{1^9}{9}$

$= \frac{511}{9}$

$$\boxed{\int_1^2 x^8\,dx = \frac{511}{9}}$$

B) $\int_0^1 8t^7 + e^t\,dt = F(1) - F(0)$

$F'(x) = f(x)$

$F'(x) = 8t^7 + e^t$

$F(x) = t^8 + \frac{e^{t+1}}{t+1}$

$= \left(1^8 + \frac{e^8}{8}\right) - \left(0^8 + \frac{e^8}{8}\right)$

$= e$

$$\boxed{\int_0^1 8t^7 + e^t\,dt = e}$$

C) $\int_{-1}^1 \frac{1}{1+x^2}\,dx = \int_{-1}^1 \arctan(x)$

$= \int_{-1}^1 1 \cdot \arctan(x)\,dx$

$= \int_{-1}^1 1 \cdot \arctan(x)\,dx = x \cdot \arctan(x) - \int_{-1}^1 x \cdot \frac{1}{1+x^2}\,dx$

$\int_{-1}^1 \frac{1}{1+x^2}\,dx = \int_{-1}^1 \frac{1}{2} \cdot \frac{1}{1+u}\,du = \frac{1}{2}\ln(1+u)$

$u = x^2$

$du = 2x\,dx$

$\Rightarrow \int_{-1}^1 \arctan(x)\,dx = x \cdot \arctan(x) - \frac{1}{2}\ln(1+x^2)$

$\int_{-1}^1 \arctan(x) = F(1) - F(-1)$

$$\boxed{\int_{-1}^1 \arctan(x)\,dx = 0}$$

D) $\int_0^{\pi/2} \sin^2(t)\,dt = \int_0^{\pi/2} \frac{1}{2}(1-\cos(2t))\,dt$

$F'(t) = f(t)$

$F'(t) = \sin^2(t)$

$F'(t) = \frac{1}{2}(1-\cos(2t))$

$F(t) = \frac{1}{2}\left(t - \frac{1}{2}\sin(2t)\right)$

$F(t) = \frac{1}{2}t - \frac{1}{4}\sin(2t)$

$\int_0^{\pi/2} \frac{1}{2}(1-\cos(2t))\,dt = F\left(\frac{\pi}{2}\right) - F(0)$

$\int_0^{\pi/2} \frac{1}{2}(1-\cos(2t))\,dt = \left(\frac{\pi}{4} - 0\right) - (0-0)$

$\int_0^{\pi/2} \frac{1}{2}(1-\cos(2t))\,dt = \frac{\pi}{4} - 0$

$$\boxed{\int_0^{\pi/2} \frac{1}{2}(1-\cos(2t))\,dt = \frac{\pi}{4}}$$

E) $\int_0^{\pi/2} \sin^4(t)\,dt = \int_0^{\pi/2} (\sin^2(t))^2\,dt$

$F'(t) = f(t)$

$F'(t) = \sin^4(t)$

$F'(t) = (\sin^2(t))^2$

$F'(t) = \left(\frac{1}{2}(1-\cos(2x))\right)^2$

$$= \int_0^{\pi/2} \left(\frac{1}{2}(1-\cos(2t))\right)^2 dt$$

$$= \frac{1}{4}\int_0^{\pi/2} (1-\cos(2t))^2\,dt$$

$$= \frac{1}{4}\int_0^{\pi/2} 1 - 2\cos(2t) + \cos^2(2t)\,dt$$

$\frac{1}{2}(1+\cos(2t))$

$\frac{1}{2} + \frac{1}{2}\cos(4t)$

$$= \frac{1}{4}\int_0^{\pi/2} \frac{3}{2} - 2\cos(2t) + \frac{1}{2}\cos(4t)\,dt$$

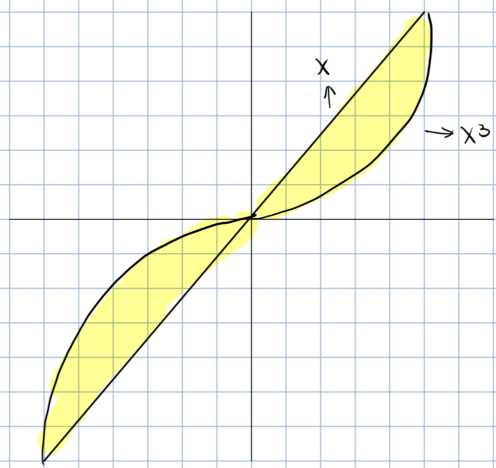
$$F = \frac{1}{4}\left(\frac{3}{2}t - \frac{2}{2}\sin(2t) + \frac{1}{24}\sin(4t)\right)$$

$$F = \frac{3}{8}t - \frac{1}{4}\sin(2t) + \frac{1}{3^2}\sin(4t)$$

$$\int_0^{\pi/2} = F\left(\frac{\pi}{2}\right) - F(0)$$

$$\boxed{\int_0^{\pi/2} \sin^4(t)\,dt = \frac{3\pi}{16}}$$

## * PROBLEM 17.2




$$2\int_0^1 x - x^3\,dx = 2\left(\frac{x^2}{2} - \frac{x^4}{4}\right)$$

$$2\int_0^1 x - x^3\,dx = 2\left(\frac{1}{2} - \frac{1}{4}\right) = \boxed{\frac{1}{2}}$$

## * PROBLEM 17.4

A) $$\int_0^1 \frac{3}{2}\sqrt{1+x}\,dx = \int_0^1 \frac{3\sqrt{1+x}}{2}\,dx$$ $F'(x) = \frac{3\sqrt{x+1}}{2}$

$$= \frac{3}{2}\int_0^1 \sqrt{u}\,du = \frac{3}{2}\int_0^1 u^{\frac{1}{2}}\,du$$ $F'(x) = \frac{3\sqrt{u}}{2}$, $u = x+1$

$$= \frac{3}{2}\int_0^1 \frac{2u^{\frac{3}{2}}}{3}\,du = \int_0^1 (x+1)^{\frac{3}{2}}\,dx$$ $F(x) = (x+1)^{\frac{3}{2}}$

$$\int_0^1 \frac{3}{2}\sqrt{x+1}\,dx = \boxed{\frac{2^{\frac{5}{2}} - 2}{2} = 1,83}$$

B) $$\int_0^{\sqrt{\log(2)}} 16xe^{-x^2}\,dx = -8\int_0^{\sqrt{\log(2)}} e^u\,du$$ $F'(x) = 16xe^{-x^2}$

$F'(x) = -\frac{1}{2x}\,du$ $u = -x^2$

$$\int_0^{\sqrt{\log(2)}} 16xe^{-x^2}\,dx = \int_0^{\sqrt{\log(2)}} -8e^u$$

$$\int_0^{\sqrt{\log(2)}} 16xe^{-x^2}\,dx = \int_0^{\sqrt{\log(2)}} -8e^{-x^2}$$

$$\int_0^{\sqrt{\log(2)}} 16xe^{-x^2}\,dx = \boxed{4}$$

C) $\int_0^{\pi} \sin^4(x)\, dx = \int_0^{\pi} (\sin^2(x))^2\, dx$

$F'(t) = f(t)$
$F'(t) = \sin^4(t)$
$F'(t) = (\sin^2(t))^2$
$F'(t) = \left(\frac{1}{2}(1-\cos(2x))\right)^2$

$$= \int_0^{\pi} \left(\frac{1}{2}(1-\cos(2x))\right)^2 dx$$

$$= \frac{1}{4}\int_0^{\pi} (1-\cos(2x))^2\, dx$$

$$= \frac{1}{4}\int_0^{\pi} 1 - 2\cos(2x) + \cos^2(2x)\, dx$$

$$\frac{1}{2}(1+\cos(2x))$$

$$\frac{1}{2} + \frac{1}{2}\cos(4x)$$

$$= \frac{1}{4}\int_0^{\pi} \frac{3}{2} - 2\cos(2x) + \frac{1}{2}\cos(4x)\, dx$$

$$\int_0^{\pi} \sin^4(x)\, dx = \boxed{\frac{3\pi}{8}}$$

D) $\int_1^{e} \frac{5\log(x)}{x}\, dx = 5\int_1^{e} u\, du$ $\quad u = \ln(x)$

$$= \int_1^{e} u\, du = \int_1^{e} \frac{5u^2}{2} = \int_1^{e} \frac{5\ln^2(x)}{2} = \frac{5}{2}$$

$$\boxed{\int_1^{e} \frac{5\log(x)}{x}\, dx = \frac{5}{2}}$$

* PROBLEM 17. 5

$u = x-1$

A) $\int_0^{3} |x-1|\, dx = \int_0^{3} |u|\, du = \frac{u}{|u|}\int_0^{3} |u| \cdot \frac{|u|}{u}\, du$

$$= \frac{u}{|u|}\int_0^{3} u\, du$$

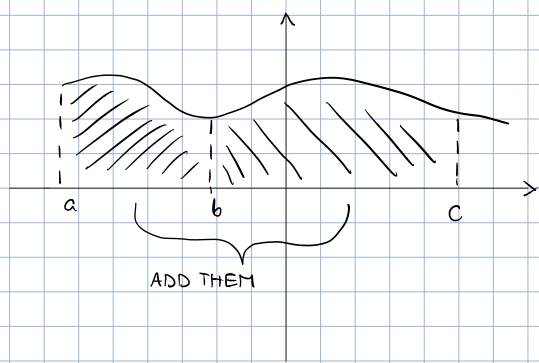
$$= \int_0^{3} \frac{u|u|}{2} = \int_0^{3} \frac{|x-1|(x-1)}{2}\, dx = \boxed{\frac{5}{2}}$$

B) $\int_0^3 |x-|x-1|| = \frac{x-|x-1|}{|x-|x-1|} \int_0^3 (x-|x-1|)\,dx = \int_0^3 \frac{(2x-1)(x(x-|x-1|-1)+1)}{2|x-|x-1||} = \boxed{2.5}$

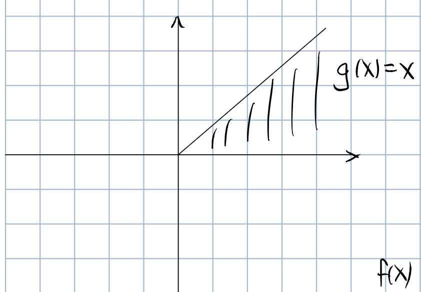
=> ASSUME POSITIVE FACTORS

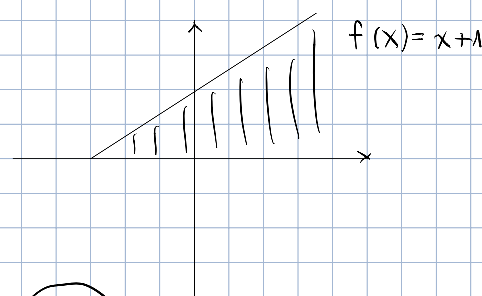
* PROBLEM 17.3

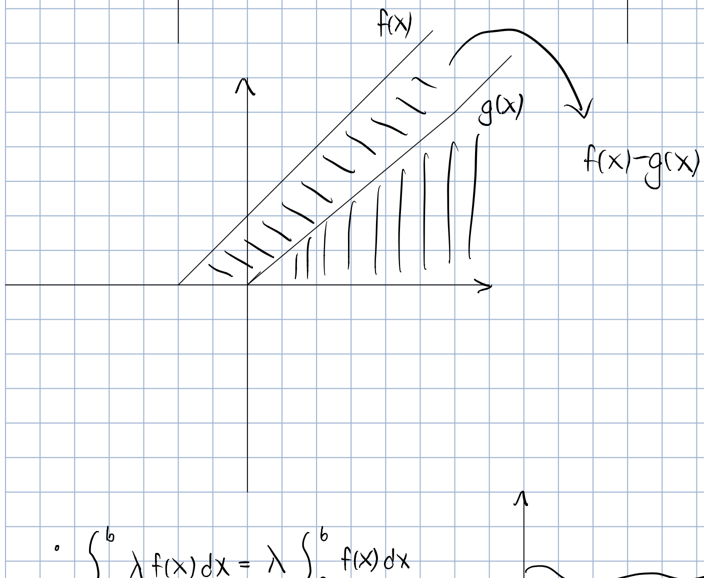
- $\int_a^b f(x)\,dx + \int_b^c f(x)\,dx = \int_a^c f(x)\,dx$




- $\int_a^b f(x)\,dx - \int_b^c f(x)\,dx = \int_a^b f(x) - g(x)\,dx$




- $\int_a^b \lambda f(x)\,dx = \lambda \int_a^b f(x)\,dx$