MATH 1A - PROBLEM SET 18

$$\int_a^b f(x)\,dx = F(b) - F(a)$$

$$F'(x) = f(x)$$

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* PROBLEM 18.1

A) $\int_0^2 4x^3 + 10x\,dx = F(2) - F(0)$

$\int_0^2 4x^3 + 10x\,dx = (2^4 + 5 \cdot 2^2) - (0^4 + 5 \cdot 0^2)$

$\int_0^2 4x^3 + 10x\,dx = \boxed{36}$

$F'(x) = 4x^3 + 10x$

$F(x) = 4 \cdot \frac{x^4}{4} + 10 \cdot \frac{x^2}{2}$

$F(x) = x^4 + 5x^2$

B) $\int_0^1 (x+1)^3\,dx = F(1) - F(0)$

$\int_0^1 (x+1)^3\,dx = \left(\frac{1}{4} - 1 + \frac{3}{2} - 1\right)$

$\int_0^1 (x+1)^3\,dx = \boxed{-\frac{1}{4}}$

$F'(x) = (x+1)^3$

$F'(x) = x^3 - 3x^2 + 3x - 1$

$F(x) = \frac{x^4}{4} - 3 \cdot \frac{x^3}{3} + 3 \cdot \frac{x^2}{2} - x$

$F(x) = \frac{x^4}{4} - x^3 + \frac{3x^2}{2} - x$

* PROBLEM 18.2

A) $\int_2^3 \frac{5}{x-1}\,dx = F(3) - F(2)$

$\int_2^3 \frac{5}{x-1}\,dx = 5\log(2) - 5\log(1)$

$\int_2^3 \frac{5}{x-1}\,dx = \boxed{5\log(2) = 3.4657}$

$F'(x) = \frac{5}{x-1}$

$F(x) = 5\log(x-1)$

B) $\int_0^{\sqrt{\pi}} \sin(x^2)\,x\,dx = F(\sqrt{\pi}) - F(0)$

$\int_0^{\sqrt{\pi}} \sin(x^2)\,x\,dx = \frac{-\cos(\pi)}{2} + \frac{\cos(0)}{2}$

$\int_0^{\sqrt{\pi}} \sin(x^2)\,x\,dx = \boxed{1}$

$F'(x) = \sin(x^2)x$

$F'(x) = \sin$

$F(x) = -\frac{\cos(u)}{2}$

$F(x) = -\frac{\cos(x^2)}{2}$

$u = x^2$

$\frac{du}{dx} = 2x$

$dx = \frac{1}{2}u$

* PROBLEM 18.3

A) $\int_1^2 2^x\,dx = F(2) - F(1)$

$F'(x) = 2^x$

$F(x) = \dfrac{2^x}{\ln(2)}$

$\int_1^2 2^x\,dx = \dfrac{4}{\ln(2)} - \dfrac{2}{\ln(2)}$

$\int_1^2 2^x\,dx = \boxed{\dfrac{2}{\ln(2)} = 2.8854}$

B) $\int_0^1 \dfrac{1}{1+x^2}\,dx = F(1) - F(0)$

$F'(x) = \dfrac{1}{1+x^2}$

$F'(x) = \arctan(x)$

$F'(x) = \arctan(x)\cdot 1$

$f = \arctan(x) \qquad g' = 1$

$f' = \dfrac{1}{x^2+1} \qquad g = x$

$= x\arctan(x) - \int \dfrac{x}{x^2+1}\,dx$

$u = x^2+1 \rightarrow \dfrac{du}{dx} = 2x$

$dx = \dfrac{1}{2}\dfrac{du}{x}$

$= \dfrac{1}{2}\int \dfrac{1}{u}\,du = \dfrac{\log(u)}{2} = \dfrac{\log(x^2+1)}{2}$

$F(x) = x\arctan(x) - \dfrac{\log(x^2+1)}{2}$

$\int_0^1 \dfrac{1}{1+x^2}\,dx = \arctan(1) - \dfrac{\ln(2)}{2}$

$\int_0^1 \dfrac{1}{1+x^2}\,dx = \boxed{\dfrac{2\ln(2) - \pi}{4} = 0.4388}$

* PROBLEM 18.4

A) $F(x) = \int_0^{x^3} \sin(t)\,dt = F(x^3) - F(0)$

$G'(t) = \sin(t)$

$G(t) = -\cos(t)$

$\int_0^{x^3} \sin(t)\,dt = -\cos(x^3) + \cos(0)$

$\int_0^{x^3} \sin(t)\,dt = 1 - \cos(x^3)$

$$\boxed{\begin{array}{l} F(x) = 1 - \cos(x^3) \\ F'(x) = 3x^2\sin(x^3) \end{array}}$$

B) $G(x) = \int_{\sin(x)}^{\cos(x)} \exp(t)\,dt$

$F'(t) = \exp(t)$
$F(t) = \exp(t)$

$$\int_{\sin(x)}^{\cos(x)} \exp(t)\,dt = F(\cos(x)) - F(\sin(x))$$

$$\int_{\sin(x)}^{\cos(x)} \exp(t)\,dt = e^{\cos(x)} - e^{\sin(x)}$$

$$G(x) = e^{\cos(x)} - e^{\sin(x)}$$

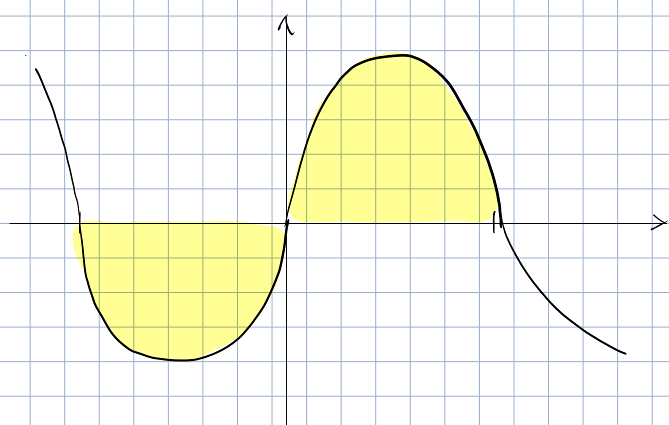
$$G'(x) = \frac{d}{dx}\left(e^{\cos(x)}\right) - \frac{d}{dx}\left(e^{\sin(x)}\right)$$

$\Rightarrow$ USING THE CHAIN RULE $u = \cos(x)$ $u = \sin(x)$

$$\boxed{G'(x) = -e^{\sin(x)} \cdot \cos(x) - \sin(x) \cdot e^{\cos(x)}}$$

* PROBLEM 18.5

A) $\int_{-\pi}^{\pi} \sin(\sin(\sin(\sin(\sin(x))))) \, dx = 0$



B) $\int_{e}^{e^x} \frac{1}{\log(x)\,x} = F(e^x) - F(e)$

$F'(x) = \frac{1}{\log(x)\,x}$

$\int_{e}^{e^x} \frac{1}{\log(x)\,x} = \log(\log(e^x)) - \log(\log(e))$

$F'(x) = \frac{1}{x\log(x)}$ $u = \log(x)$, $du = \frac{1}{x}dx$

$\int_{e}^{e^x} \frac{1}{\log(x)\,x} = \boxed{\log(\log(e^x))}$

$= \int \frac{1}{u}\,du = \log(u)$

$F(x) = \log(\log(x))$