MATH 1A - PROBLEM SET 20

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* PROBLEM 20.1

$f(x) = 2x^5 - 24x$

$g(x) = 4x^2$

$x > 0$

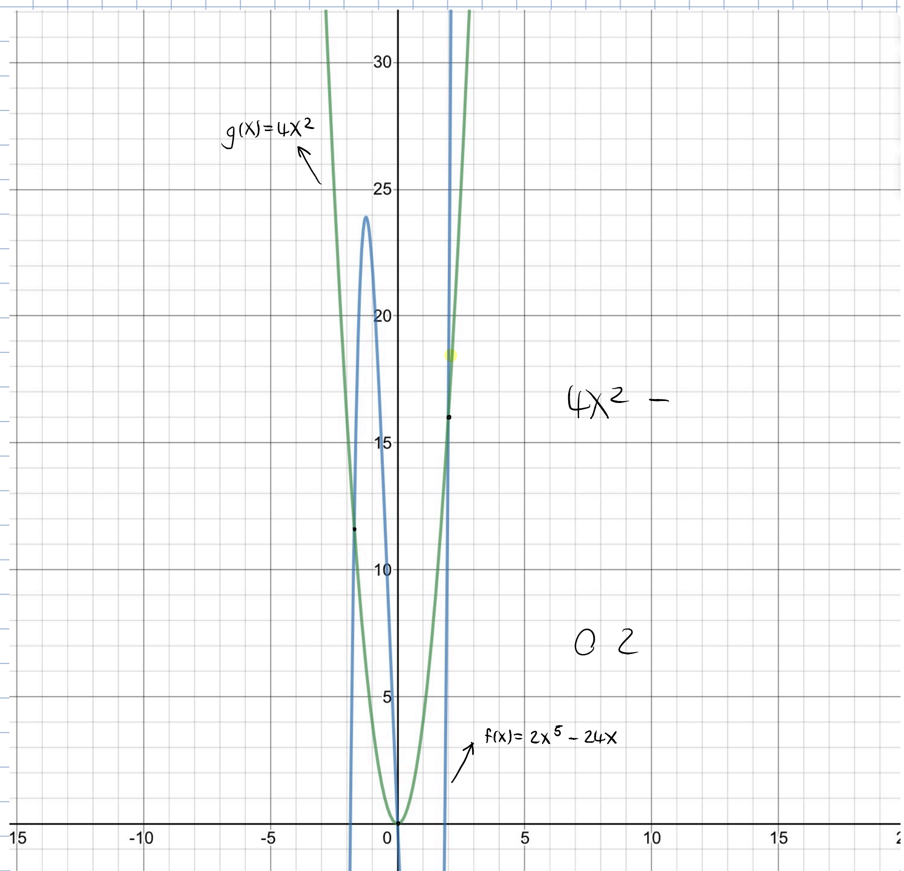
$f(x) = g(x)$

$2x^5 - 24x = 4x^2$

$x_1 = 0$

$x_2 = 2$

$x_3 = -1.7113$



$$\int_0^2 4x^2 - (2x^5 - 24x) = \int_0^2 4x^2 - 2x^5 + 24x$$

$$= 4\frac{x^3}{3} - 2\frac{x^6}{6} + 24\frac{x^2}{2}\Big|_0^2$$

$$= 4\cdot\frac{8}{3} - 2\cdot\frac{64}{6} + 24\cdot\frac{4}{2}$$

$$\int_0^2 4x^2 - (2x^5 - 24x) = \boxed{\frac{112}{3}}$$

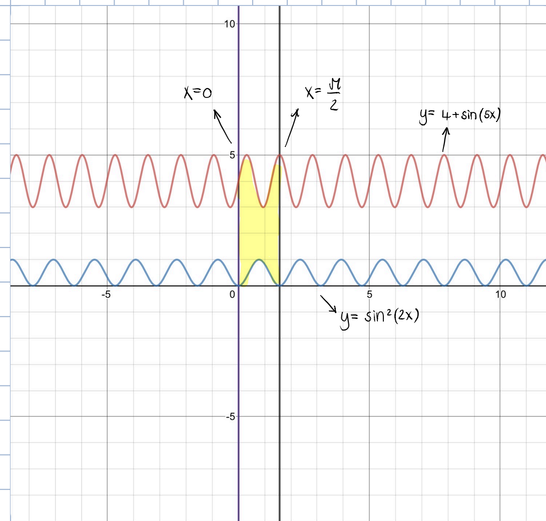
* PROBLEM 20.2

$x = 0$

$x = \frac{\pi}{2}$

$y = 4 + \sin(5x)$

$y = \sin^2(2x)$



$$A = \int_0^{\frac{\pi}{2}} 4 + \sin(5x) - \sin^2(2x)\, dx$$

$$\int_0^{\frac{\pi}{2}} \sin(5x) - \sin^2(x) + 4 \, dx = \overset{①}{\int_0^{\frac{\pi}{2}} \sin(5x)\,dx} - \overset{②}{\int_0^{\frac{\pi}{2}} \sin^2(2x)\,dx} + \overset{③}{4\int_0^{\frac{\pi}{2}} 1\,dx}$$

$$\int_0^{\frac{\pi}{2}} \sin(5x) - \sin^2(x) + 4 \, dx = -\frac{\cos(5x)}{5} + \frac{\cos(2x)\sin(2x)}{4} + 4x$$

$$\int_0^{\frac{\pi}{2}} \sin(5x) - \sin^2(x) + 4 \, dx = \frac{1}{20}\left(70x + 5\sin(2x) - 4\cos(5x)\right)\Big|_0^{\frac{\pi}{2}}$$

$$\int_0^{\frac{\pi}{2}} \sin(5x) - \sin^2(x) + 4 \, dx = \boxed{\frac{1}{5} + \frac{7\pi}{4} \approx 5.6978}$$

① $\Rightarrow \int_0^{\frac{\pi}{2}} \sin(5x)\,dx \quad u = 5x \rightarrow \frac{du}{dx} = 5 \rightarrow dx = \frac{1}{5}du$

$$= \frac{1}{5}\int_0^{\frac{\pi}{2}} \sin(u)\,du = -\frac{\cos(u)}{5} = \frac{-\cos(5x)}{5}$$

③ $\Rightarrow \int 1\,dx = x$

② $\Rightarrow \int_0^{\frac{\pi}{2}} \sin^2(2x)\,dx \quad u = 2x \rightarrow \frac{du}{dx} = 2 \rightarrow dx = \frac{1}{2}du$

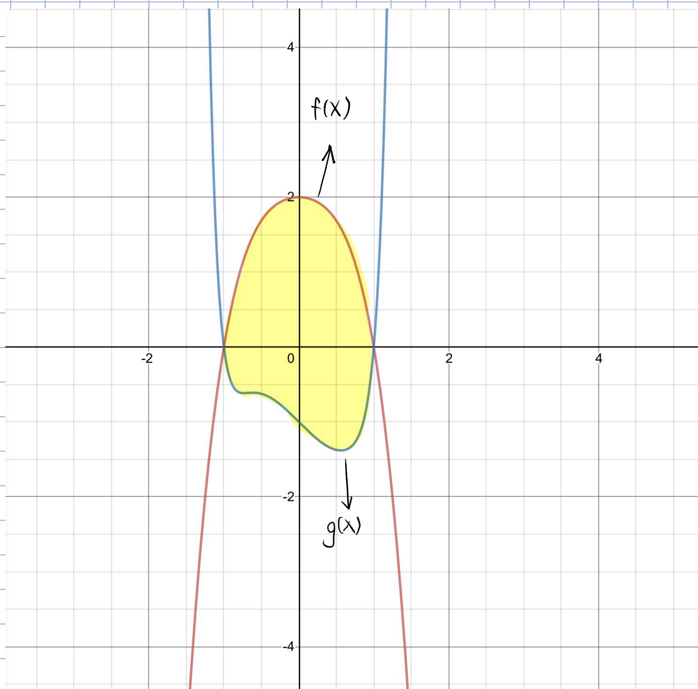
$\frac{1}{2}\int_0^{\frac{\pi}{2}} \sin^2(u)\,du \Rightarrow$ USING THE FORMULA $\int \sin^n(u)\,du = \frac{n-1}{n}\int \sin^{n-2}(u)\,du - \frac{\cos(u)\sin^{n-1}(u)}{n}$

$$= -\frac{\cos(u)\sin(u)}{2} + \frac{1}{2}\int 1\,du = \frac{u}{4} - \frac{\cos(u)\sin(u)}{4} = \frac{x}{2} - \frac{\cos(2x)\sin(2x)}{4}$$

* PROBLEM 20.3

$f(x) = 1 - x^4 - x^2 + 1$

$g(x) = x^{10} - 1 + x^3 - x$



$$\int_a^b g(x) - f(x)\,dx = \int_{-1}^{1} (1 - x^4 - x^2 + 1) - (x^{10} - 1 + x^3 - x)\,dx$$

$$= \int_{-1}^{1} 1 - x^4 - x^2 + 1 - x^{10} + 1 - x^3 + x\,dx$$

$$= \int_{-1}^{1} -x^{10} - x^4 - x^3 - x^2 + x + 3\,dx$$

$$= -\frac{x^{11}}{11} - \frac{x^5}{5} - \frac{x^4}{4} - \frac{x^3}{3} + \frac{x^2}{2} + 3 \cdot \frac{x}{1} \Big|_{-1}^{1}$$

$$= \left(-\frac{1}{11} - \frac{1}{5} - \frac{1}{4} - \frac{1}{3} + \frac{1}{2} + 3\right) - \left(\frac{1}{11} + \frac{1}{5} - \frac{1}{4} + \frac{1}{3} + \frac{1}{2} - 3\right)$$

$$= \frac{1733}{660} + \frac{1403}{660} = \boxed{\frac{784}{165} = 4.75}$$

$$\int_{-1}^{1} (1 - x^4 - x^2 + 1) - (x^{10} - 1 + x^3 - x) = \boxed{\frac{784}{165} = 4.75}$$

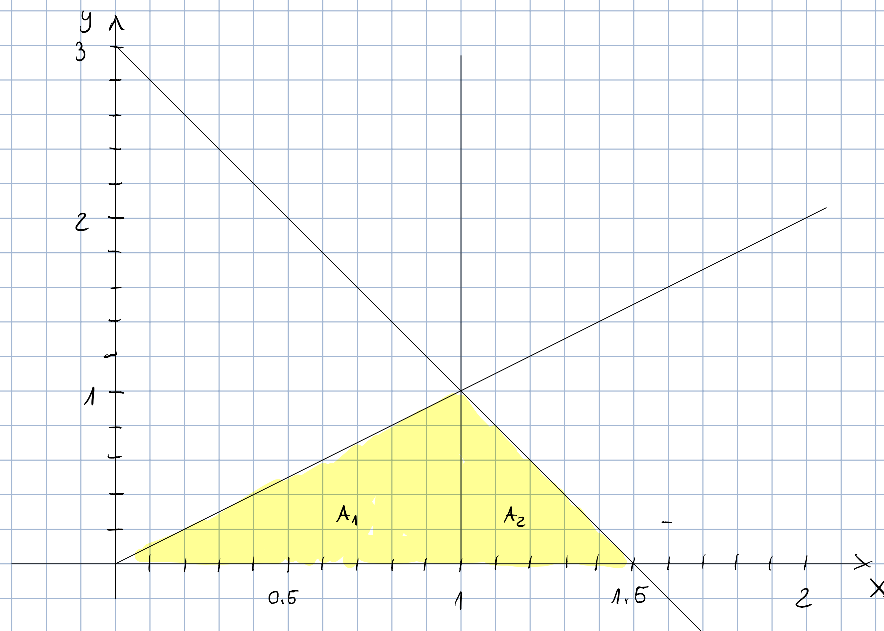
* PROBLEM 20.4

$y = x$

$y = 3 - 2x$

$y = 0$

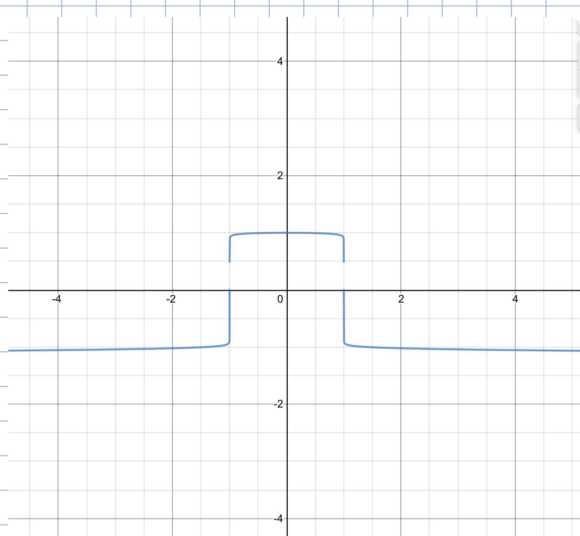
$\boxed{A = A_1 + A_2}$



$A = \dfrac{1,5 \cdot 1}{2} = \boxed{\dfrac{3}{4}}$ => OR WE COULD SOLVE BY SPLITING THE TRIANGLE IN TWO PARTS AND THEN INTEGRATING

* PROBLEM 20.5

$x^2 + |y|^{|51|} \leq 1$



$2\int_{-1}^{1} \underbrace{(1-x^2)^{1/51}}_{\text{FOR ABS. VALUE}} dx \simeq \boxed{3.9526}$