MATH 1A - PROBLEM SET 23

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* PROBLEM 1

$f(x) = 5e^{-5x}$

$x = 1$

$x = 4$

$$\int_1^4 f(x)\,dx = \int_1^4 5e^{-5x}\,dx$$

$$= \int_1^4 5e^{u}\,du \qquad u = -5x$$

$$= -\int_1^4 e^{u}\,du = -e^{u}\Big|_1^4$$

$$= -e^{-5x}\Big|_1^4 = -e^{-20} + e^{-5}$$

$$= e^{-20}\left(e^{15} - 1\right)$$

$$\boxed{= 0.006737945}$$

* PROBLEM 2

$f(x) = \frac{1}{4}e^{-\frac{x}{4}}$

$x = 1$

$x = 2$

$$\int_1^2 f(x)\,dx = \int_1^2 \frac{1}{4}e^{-\frac{x}{4}}\,dx$$

$$= \int_1^2 \frac{e^{-\frac{x}{4}}}{4}\,dx$$

$$= \int_1^2 \frac{e^{u}}{4}\,du \qquad u = -\frac{1}{4}$$

$$= -\int_1^2 e^{u}\,du = -e^{u}\Big|_1^2$$

$$= -e^{-\frac{x}{4}}\Big|_1^2 = -e^{-\frac{1}{2}} + e^{-\frac{1}{4}}$$

$$= \boxed{0.17227}$$

* PROBLEM 3

$$f(x) = \frac{1}{\pi\sqrt{1-x^2}} \qquad f(x) = 0 \quad \text{EXCEPT WHEN } f(x) = [-1, 1]$$

=> VERIFY THAT f(X) IS A PROBABILITY DISTRIBUTION

$$F'(x) = f(x)$$

$$F'(x) = \frac{1}{\pi\sqrt{1-x^2}}$$

$$F'(x) = \frac{1}{\pi} \cdot \arcsin(x)$$

$$F(x) = \int \frac{1}{\pi} \cdot \arcsin(x)$$

$$F(x) = \frac{1}{\pi} \int \arcsin(x)$$

$$F(x) = \boxed{\frac{\arcsin(x)}{\pi}}$$

* PROBLEM 4

$>1 \qquad 1, \infty$

CAUCHY DISTRIBUTION: $\boxed{f(x) = \frac{1}{\pi} \frac{1}{1+x^2}}$ SYMETRICAL SO MULTIPLY BY TWO

A) ABSOLUTE VALUE LARGER THAN 1

$$2 \cdot \int_1^\infty \frac{1}{\pi} \frac{1}{1+x^2} \, dx$$

$$\frac{2}{\pi} \cdot \int_1^\infty \frac{1}{1+x^2} \, dx$$

$$2 \cdot \frac{\arctan(x)}{\pi} = 2 \cdot \frac{1}{4} = \boxed{\frac{1}{2}}$$

B) SMALLER THAN $-\frac{\sqrt{3}}{2}$

$$\int_{-\infty}^{-\frac{\sqrt{3}}{2}} \frac{1}{\pi} \frac{1}{1+x^2}\, dx$$

$$= \frac{1}{\pi} \int_{-\infty}^{-\frac{\sqrt{3}}{2}} \frac{1}{1+x^2}\, dx$$

$$= \frac{\arctan(x)}{\pi} \Bigg|_{-\infty}^{-\frac{\sqrt{3}}{2}} = \frac{\frac{\pi}{2} - \arctan\left(\frac{\sqrt{3}}{2}\right)}{\pi} = \boxed{0.2728}$$

* PROBLEM 5

MEAN OF THE DISTRIBUTION : $\boxed{\int_{-\infty}^{\infty} x f(x)\, dx}$

A) STANDARD NORMAL DISTRIBUTION

$$f(x) = \frac{1}{\sqrt{2\pi}} e^{-\frac{x^2}{2}}$$

$$\int_{-\infty}^{\infty} x f(x)\, dx = \int_{-\infty}^{\infty} x \frac{1}{\sqrt{2\pi}} e^{-\frac{x^2}{2}}\, dx$$

$$= \int_{-\infty}^{\infty} \frac{x e^{-\frac{x^2}{2}}}{\sqrt{2\pi}}\, dx$$

$$= -\frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} e^{u}\, du \quad u = \frac{-x^2}{2}$$

$$= -\frac{e^{u}}{\sqrt{2\pi}}$$

$$= \frac{-e^{-\frac{x^2}{2}}}{\sqrt{2\pi}}$$

$$\int_{-\infty}^{\infty} \frac{x e^{-\frac{x^2}{2}}}{\sqrt{2\pi}}\, dx = \boxed{0}$$

=> USING THE FUNCTIONS PROPERTY OF BEING SYMETRIC

B) CAUCHY DISTRIBUTION

$$f(x) = \frac{1}{\pi} \frac{1}{1+x^2}$$

$$\int_{-\infty}^{\infty} x f(x) = \int_{-\infty}^{\infty} x \cdot \frac{1}{\pi} \cdot \frac{1}{1+x^2} \, dx$$

$$= \int_{-\infty}^{\infty} \frac{x}{\pi(1+x^2)} \, dx$$

$$= \frac{1}{2\pi} \int \frac{1}{u} \, du \qquad u = x^2 + 1$$

$$= \frac{\ln(u)}{2\pi} = \frac{\ln(x^2+1)}{2\pi}$$

$$\int_{-\infty}^{\infty} \frac{x}{\pi(1+x^2)} \, dx = \boxed{0}$$ => THE DEFINITE INTEGRAL WAS CALCULATED USING THE FUNCTION'S PROPERTY OF BEING SYMETRIC

C) ARC-SIN DISTRIBUTION

$$f(x) = \frac{1}{\pi} \frac{1}{\sqrt{1-x^2}} \text{ ON } [-1, 1]$$

$$\int_{-1}^{1} x f(x) \, dx = \int_{-1}^{1} \frac{1}{\pi} \frac{x}{\sqrt{1-x^2}} \, dx$$

$$= \int_{-1}^{1} \frac{x}{\pi \sqrt{1-x^2}} \, dx$$

$$= \int_{-1}^{1} \frac{x}{\pi \sqrt{u}} \, du \qquad u = 1 - x^2$$

$$= -\frac{1}{2\pi} \int_{-1}^{1} \frac{1}{\sqrt{u}} \, du$$

$$= \frac{-\sqrt{u}}{\pi} \Big|_{-1}^{1} = \frac{-\sqrt{1-x^2}}{\pi} \Big|_{-1}^{1} = \boxed{0}$$