MATH 1A - PROBLEM SET 25.

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$$(uv)' = vu' + uv'$$
$$uv = \int v\,du + \int u\,dv$$
$$\int u\,dv = uv - \int v\,du$$

$$\int u(x)\,v'(x)\,dx = u(x)\,v(x) - \int u'(x)\,v(x)\,dx$$

* PROBLEM 1

$$\int x^3 \log(x)\,dx \qquad u = \log(x) \quad dv = x^3$$

$$= \log(x) \cdot \frac{x^4}{4} - \int \frac{x^4}{4} \cdot \frac{1}{x}\,dx \qquad du = \frac{1}{x} \quad v = \frac{x^4}{4}$$

$$= \frac{\log(x)\,x^4}{4} - \int \frac{x^4}{4x}\,dx$$

$$= \frac{\log(x)\,x^4}{4} - \frac{1}{4}\int x^3\,dx$$

$$= \frac{\log(x)\,x^4}{4} - \frac{1}{4} \cdot \frac{x^4}{4}$$

$$= \frac{\log(x)\,x^4}{4} - \frac{x^4}{16} + C$$

$$= \frac{x^4(4\log(x) - 1)}{16} + C$$

* PROBLEM 2

$$\int x^5 \sin(x)\, dx$$

$$-x^5 \cdot \cos(x) - \int -5x^4 \cos(x)\, dx$$

$$-x^5 \cos(x) + 5\int x^4 \cos(x)\, dx$$

$$-x^5 \cos(x) + 5\left(x^4 \sin(x) - \int 4x^3 \sin(x)\, dx \right)$$

$$-x^5 \cos(x) + 5x^4 \sin(x) - 20\int x^3 \sin(x)\, dx$$

$$-x^5 \cos(x) + 5x^4 \sin(x) - 20\left(-x^3 \cos(x) - \int -3x^2 \cos(x)\, dx \right)$$

$$-x^5 \cos(x) + 5x^4 \sin(x) + 20x^3 \cos(x) + 20\int -3x^2 \cos(x)\, dx$$

$$-x^5 \cos(x) + 5x^4 \sin(x) + 20x^3 \cos(x) - 60\int x^2 \cos(x)\, dx$$

$$-x^5 \cos(x) + 5x^4 \sin(x) + 20x^3 \cos(x) - 60\left(x^2 \sin(x) - \int 2x \sin(x)\, dx \right)$$

$$-x^5 \cos(x) + 5x^4 \sin(x) + 20x^3 \cos(x) - 60x^2 \sin(x) + 120\int x \sin(x)\, dx$$

$$-x^5 \cos(x) + 5x^4 \sin(x) + 20x^3 \cos(x) - 60x^2 \sin(x) + 120\left(-x \cos(x) - \int -\cos(x)\, dx \right)$$

$$-x^5 \cos(x) + 5x^4 \sin(x) + 20x^3 \cos(x) - 60x^2 \sin(x) - 120x \cos(x) - 120\int -\cos(x)\, dx$$

$$-x^5 \cos(x) + 5x^4 \sin(x) + 20x^3 \cos(x) - 60x^2 \sin(x) - 120x \cos(x) + 120\int \cos(x)\, dx$$

$$-x^5 \cos(x) + 5x^4 \sin(x) + 20x^3 \cos(x) - 60x^2 \sin(x) - 120x \cos(x) + 120\sin(x) + C$$

$$= \boxed{(5x^4 - 60x^2 + 120)\sin(x) + (-x^5 + 20x^3 - 120x)\cos(x) + C}$$

OR USING THE TIC-TAC-TOE METHOD

$$\int x^5 \sin(x)\,dx$$

x^5	$\sin(x)$	
$5x^4$	$-\cos(x)$	$+$
$20x^3$	$-\sin(x)$	$-$
$60x^2$	$\cos(x)$	$+$
$120x$	$\sin(x)$	$-$
120	$-\cos(x)$	$+$
0	$-\sin(x)$	$-$

$$= -x^5\cos(x) + 5x^4\sin(x) + 20x^3\cos(x) - 60x^2\sin(x) - 120x\cos(x) + 120\sin(x) + C$$

$$= \boxed{\left(5x^4 - 60x^2 + 120\right)\sin(x) + \left(-x^5 + 20x^3 - 120x\right)\cos(x) + C}$$

* PROBLEM 3

$$\int 2x^6 \exp(x)\,dx$$

$2x^6$	$\exp(x)$	
$12x^5$	$\exp(x)$	$+$
$60x^4$	$\exp(x)$	$-$
$240x^3$	$\exp(x)$	$+$
$720x^2$	$\exp(x)$	$-$
$1440x$	$\exp(x)$	$+$
1440	$\exp(x)$	$-$
0	$\exp(x)$	$+$

$$= 2x^6\exp(x) - 12x^5\exp(x) + 60x^4\exp(x) - 240x^3\exp(x) + 720x^2\exp(x) - 1440x\exp(x) + 1440\exp(x) + C$$

$$= \boxed{2e^x\left(x^6 - 6x^5 + 30x^4 - 120x^3 + 360x^2 - 720x + 720\right) + C}$$

★ PROBLEM 4

$$\int \sqrt{x}\, \log(x)\, dx$$

$$\frac{2x^{\frac{3}{2}} \log(x)}{3} - \int \frac{2x^{\frac{3}{2}}}{3} \cdot \frac{1}{x}\, dx$$

$$\frac{2x^{\frac{3}{2}} \log(x)}{3} - \int \frac{2\sqrt{x}}{3}\, dx$$

$$\frac{2x^{\frac{3}{2}} \log(x)}{3} - \frac{2}{3} \int \sqrt{x}\, dx$$

$$\frac{2x^{\frac{3}{2}} \log(x)}{3} - \frac{2}{3} \cdot \frac{2x^{\frac{3}{2}}}{3}$$

$$\boxed{= \frac{2x^{\frac{3}{2}} \log(x)}{3} - \frac{4x^{\frac{3}{2}}}{9} + C = \frac{2x^{\frac{3}{2}}(3\log(x) - 2)}{9} + C}$$

★ PROBLEM 5.

$$I = \int \sin(x) \exp(-x)\, dx$$

$$= -e^{-x} \sin(x) - \int -e^{-x} \cos(x)\, dx$$

$$= -e^{-x} \sin(x) - \left(e^{-x} \cos(x) - \int -e^{-x} \sin(x)\, dx\right)$$

$$= -e^{-x} \sin(x) - \left(e^{-x} \cos(x) + \int e^{-x} \sin(x)\, dx\right)$$

$$= -e^{-x} \sin(x) - \left(e^{-x} \cos(x) + I\right)$$

$$= -e^{-x} \sin(x) - e^{-x} \cos(x) - I + C$$

SOLVING FOR $\boxed{I = \frac{-e^{-x} \sin(x) - e^{-x} \cos(x)}{2} + C = \frac{e^{-x}(\sin(x) + \cos(x))}{2} + C}$