MATH 1A - PROBLEM SET 27

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* PROBLEM 27.1

$x_0 = 0$

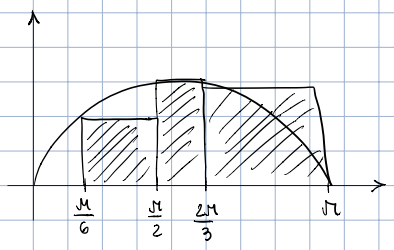
$x_1 = \pi/6$

$x_2 = \pi/2$

$x_3 = 2\pi/3$

$$\int_0^{\pi} \sin(x)\,dx = -\cos(x)\Big|_0^{\pi} = 1 + 1 = \boxed{2}$$

$$\left(0 \cdot \frac{\pi}{6} + \frac{1}{2} \cdot \frac{\pi}{3} + 1 \cdot \frac{\pi}{6} + \frac{\sqrt{3}}{2} \cdot \frac{\pi}{3}\right) = \boxed{1.95}$$



* PROBLEM 27.2

$[0, 1]$

$x_1 = 0.123$ $\Rightarrow$ SUM OF CUBES = 2.3767 $\quad 2.3767 / 10 = \boxed{0.23767}$

$x_2 = 0.756$

$x_3 = 0.532$

$x_4 = 0.378$

$x_5 = 0.644$

$x_6 = 0.917$

$x_7 = 0.43$

$x_8 = 0.289$

$x_9 = 0.824$

$x_{10} = 0.333$

$$\int_0^1 x^3\,dx = \frac{x^4}{4}\Big|_0^1 = \boxed{0.25}$$

* PROBLEM 27.3

$$\frac{\left[f(a) + 4f((a+b)/2) + f(b)\right]}{6}(b-a)$$

$f(x) = \sin(x)$

① $[a, b] = [0, \pi/2]$

$$\frac{\left[f(a) + 4f((a+b)/2) + f(b)\right]}{6}(b-a)$$

$$\frac{\sin(0) + 4\sin(\pi/4) + \sin(\pi/2)}{6} \cdot \frac{\pi}{2}$$

$$\frac{0 + 2\sqrt{2} + 1}{6} \cdot \frac{\pi}{2} = \frac{\pi(2\sqrt{2}+1)}{12}$$

② $[a, b] = [\pi/2, \pi]$

$$\frac{\left[f(a) + 4f((a+b)/2) + f(b)\right]}{6}(b-a)$$

$$\frac{\sin(\pi/2) + 4\sin(3\pi/4) + \sin(\pi)}{6}(b-a)$$

$$\frac{1 + 2\sqrt{2} + 0}{6} \cdot \frac{\pi}{2} = \frac{\pi(1+2\sqrt{2})}{12}$$

$$\frac{\pi(2\sqrt{2}+1)}{12} + \frac{\pi(2\sqrt{2}+1)}{12} = \frac{\pi(2\sqrt{2}+1)}{6} = \boxed{2.004}$$

$$\int_0^{\pi} \sin(x)\,dx = -\cos(x)\Big|_0^{\pi} = 1 + 1 = \boxed{2}$$

✳ PROBLEM 27.5

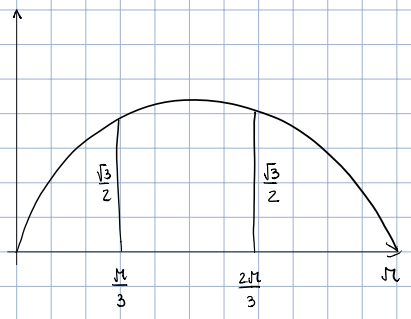
A) $\int_0^1 \sin\left(\frac{1}{x^2}\right)\frac{1}{x^2}\,dx =$ **0.316**

I USED WOLFRAM ALPHA

B) $\int_0^1 \sin\left(\frac{1}{x^2}\right)\frac{1}{x^4}\,dx =$ **DOES NOT CONVERGE**

✳ PROBLEM 27.4

$[a,b] = [0,\pi]$ $f(x) = \sin(x)$



$$\int_0^{\pi} \sin(x)\,dx = -\cos(x)\Big|_0^{\pi} = 1 + 1 = 2$$

$$\frac{0 + \frac{\sqrt{3}}{2}\cdot 3 + \frac{\sqrt{3}}{2}\cdot 3 + 0}{8}\cdot \pi = 2.0405$$