MATH 1A - PROBLEM SET (29)

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* PROBLEM 29.1

$$\int \sqrt{1-9x^2}\, dx \qquad x = \frac{\sin u}{3}$$

$$u = \arcsin(3x)$$

$$dx = \frac{\cos(u)\, du}{3}$$

$$\int \frac{\cos(u)\sqrt{1-\sin^2(u)}}{3}\, du$$

$$\frac{1}{3}\int \cos^2(u)\, du$$

$$= \frac{1}{3}\left(\frac{\cos(u)\sin(u)}{2} + \frac{1}{2}\int 1\, du\right)$$

$$= \frac{1}{3}\left(\frac{\cos(u)\sin(u)}{2} + \frac{u}{2}\right)$$

$$= \frac{\cos(u)\sin(u)}{6} + \frac{u}{6}$$

$$= \boxed{\frac{\arcsin(3x)}{6} + \frac{\sqrt{1-9x^2}}{2} + C}$$

* PROBLEM 29.2

$$\int (1-x^2)^{3/2}\, dx \qquad x = \sin(u)$$

$$u = \arcsin(x)$$

$$dx = \cos(u)\, du$$

$$\int \cos^4(u)\, du$$

$$= \frac{\cos^3(u)\sin(u)}{4} + \frac{3}{4}\int \cos^2(u)\, du$$

$$= \frac{\cos^3(u)\sin(u)}{4} + \frac{3}{4}\left(\frac{\cos(u)\sin(u)}{2} + \frac{1}{2}\int 1\, du\right)$$

$$= \frac{\cos^3(u)\sin(u)}{4} + \frac{3\cos(u)\sin(u)}{8} + \frac{3u}{8} + C$$

$$= \boxed{\frac{3\arcsin(x)}{8} + \frac{x(1-x^2)^{3/2}}{4} + \frac{3x\sqrt{1-x^2}}{8} + C}$$

* PROBLEM 29.3

$$\int \frac{\sqrt{1-x^2}}{x^2}\,dx$$

$$\int \sqrt{1-x^2}\cdot\frac{1}{x^2}\,dx$$

$$= -\frac{\sqrt{1-x^2}}{x} - \int \frac{1}{\sqrt{1-x^2}}\,dx$$

$$= -\frac{\sqrt{1-x^2}}{x} - \arcsin(x)$$

$$= \boxed{-\arcsin(x) - \frac{\sqrt{1-x^2}}{x} + C}$$

$$\int fg' = fg - \int f'g$$

$$f = \sqrt{1-x^2} \qquad g' = \frac{1}{x^2}$$

$$f' = -\frac{x}{\sqrt{1-x^2}} \qquad g = -\frac{1}{x}$$

* PROBLEM 29.4

$$\int \frac{1}{1+\sin(x)}\,dx$$

$$u = \tan(x/2)$$

$$dx = \frac{2du}{1+u^2}$$

$$\sin(x) = \frac{2u}{1+u^2}$$

$$\cos(x) = \frac{1-u^2}{1+u^2}$$

$$\int \frac{1}{1+\frac{2u}{1+u^2}}\cdot\frac{2du}{1+u^2}$$

$$\int \frac{1}{\frac{1+u^2+2u}{1+u^2}}\cdot\frac{2du}{1+u^2}$$

$$\int \frac{1+u^2}{1+u^2+2u}\cdot\frac{2du}{1+u^2}$$

$$\int \frac{2}{1+2u+u^2}\,du$$

$$2\int \frac{1}{(u+1)^2}\,du$$

$$= 2\cdot\left(-\frac{1}{u+1}\right)$$

$$= -\frac{2}{u+1}$$

$$= \boxed{-\frac{2}{\tan\left(\frac{x}{2}\right)+1} + C}$$

* PROBLEM 29.5

$$\int_0^{\pi/3} \frac{1}{\cos(x)}\,dx$$

$$u = \tan\left(\frac{x}{2}\right)$$

$$dx = \frac{2\,du}{1+u^2}$$

$$\sin(x) = \frac{2u}{1+u^2}$$

$$\cos(u) = \frac{1-u^2}{1+u^2}$$

NEW BOUNDS

$$u = \tan\left(\frac{x}{2}\right)$$

$$u = \tan\left(\frac{\pi}{3}/2\right) \qquad u = \tan(0)$$

$$u = \frac{\sqrt{3}}{3} \qquad u = 0$$

$$\int_0^{\pi/3} \frac{2\,du}{1+u^2} \cdot \frac{1+u^2}{1-u^2}$$

$$\int_0^{\pi/3} \frac{2\,du}{1-u^2}$$

$$\int_0^{\pi/3} \frac{1}{1-u} + \frac{1}{1+u}\,du$$

$$= -\log(1-u) + \log(1+u) \Big|_0^{\frac{\sqrt{3}}{3}}$$

$$= -\log\left(1-\frac{\sqrt{3}}{3}\right) + \log\left(1+\frac{\sqrt{3}}{3}\right) + \log(1) - \log(1)$$

$$= \boxed{1.31695}$$