MATH 1A - PROBLEM SET (31.)

* PROBLEM 31.1

A) $f(x)$ = MARGINAL COST
$g(x)$ = AVERAGE COST
$F(x)$ = TOTAL COST

$f(x) = \frac{1}{x}$

$F(x) = \int \frac{1}{x}$

$F(x) = \log(x)$

$g(x) = \frac{F(x)}{x}$

$g(x) = \frac{\log(x)}{x}$

BREAK-EVEN POINT

$f(x) = g(x)$

$\frac{1}{x} = \frac{\log(x)}{x}$

$\log(x) = 1$

$\boxed{x = e}$

B) $f(x)$ = MARGINAL COST
$g(x)$ = AVERAGE COST
$F(x)$ = TOTAL COST

$f(x) = x^7$

$F(x) = \frac{x^8}{8}$

$g(x) = \frac{F(x)}{x} = \frac{\frac{x^8}{8}}{\frac{x}{1}} = \frac{x^8}{8x} = \frac{x^7}{8}$

$\Rightarrow \quad 8g(x) = f(x)$

$8 \cdot \frac{x^7}{8} = x^7$

$\boxed{x^7 = x^7}$

$\Rightarrow$ $x = 0$ IS THE ONLY BREAK-EVEN POINT

$f(x) = g(x)$

$x^7 = \frac{x^7}{8}$

$x^7 = 0$

$\boxed{x = 0}$

* PROBLEM 31.2

$f(x) = \cos(x)$
$F(x) =$
$g(x) =$
$f = g$
$g' = 0$

$f(x) = \cos(x)$
$F(x) = \sin(x)$

$g(x) = \frac{F(x)}{x} = \frac{\sin(x)}{x}$

$g(x) = \frac{\sin(x)}{x}$

$g'(x) = \frac{x\cos(x) - \sin(x)}{x^2}$

$\boxed{g'(x) = 0}$

$\frac{x\cos(x) - \sin(x)}{x^2} = 0$

$x\cos(x) = \sin(x)$

$\boxed{\cos(x) = \frac{\sin(x)}{x}}$

$\boxed{f = g}$

$\boxed{\cos(x) = \frac{\sin(x)}{x}}$

* PROBLEM 31.3

$Q(L) = 5000L^3 - 3L^5$

$Q'(L) = 15\,000L^2 - 15L^4$

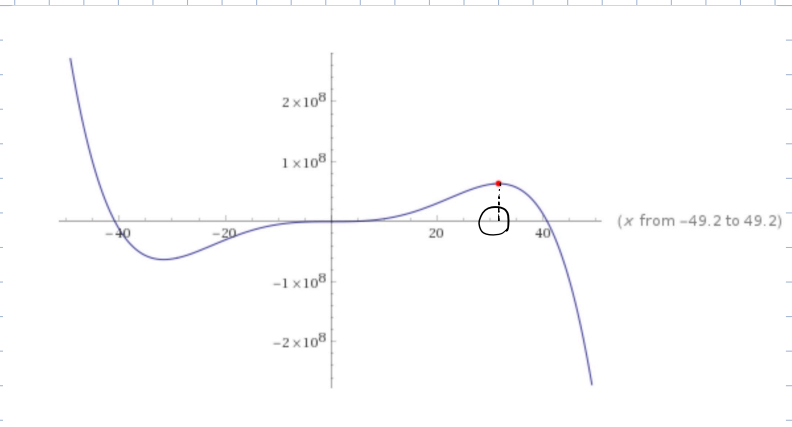
$Q'(L) = 0$

$15\,000L^2 - 15L^4 = 0$

$x_1 = 10\sqrt{10}$

$x_2 = 0$

$x_3 = -10\sqrt{10}$



=> SINCE LABOR CAN'T BE NEGATIVE AND WHEN $L=0$ THERE IS NO PRODUCTIVITY WE CAN CONCLUDE THAT $L = 10\sqrt{10}$ GIVES THE MAXIMAL PRODUCTION

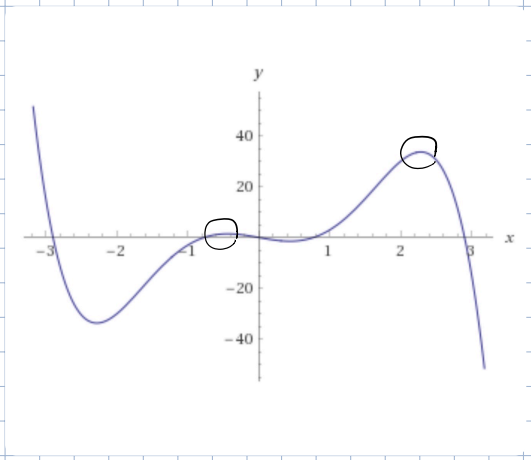
* PROBLEM 31.4

$F(X)$ = TOTAL REVENUE

$F(X) = -5X - X^5 + 9X^3$

$F'(X) = -5 - 5X^4 + 27X^2$

$F''(X) = -20X^3 + 54X$



$F'(X) = 0$

$-5 - 5X^4 + 27X^2 = 0$

$-5X^4 + 27X^2 - 5 = 0$

$X_1 = 2.2821$ — $F''(2.2821) = -114.4692$ => LOCAL MAXIMUM

$X_2 = 0.43819$ — $F''(0.43819) = 21.9795$ => LOCAL MINIMUM

$X_3 = -0.43819$ — $F''(-0.43819) = -21.9795$ => LOCAL MAXIMUM

$X_4 = -2.2821$ — $F''(-2.2821) = 114.4692$ => LOCAL MINIMUM

TOTAL REVENUE HAS A LOCAL MAXIMUM AT $X = 2.2821$, $X = -0.43819$

* PROBLEM 31.5

$y = mx$

$(3,4)$ $\quad f(m) = (3m-4)^2 + (6m-3)^2 + (2m-5)^2$

$(6,3)$ $\quad f(m) = 9m^2 - 24m + 16 + 36m^2 - 36m + 9 + 4m^2 - 20m + 25$

$(2,5)$ $\quad f(m) = 49m^2 - 80m + 50$

$f'(m) = 98m - 80$

$98m = 80$

$$m = \frac{40}{49} \quad \Rightarrow \quad y = \frac{40}{49}x$$