MATH 1A - PROBLEM SET 32.

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* PROBLEM 32.1

1. $\log(\sin(\exp(x)))$
2. $\log(\exp(\sin(x)))$
3. $\exp(\log(\sin(x)))$
4. $\exp(\sin(\log(x)))$
5. $\sin(\log(\exp(x)))$
6. $\sin(\exp(\log(x)))$

1. $\log(\sin(\exp(x)))$

$$\frac{d}{dx}(\log(\sin(\exp(x)))$$

$$= \frac{1}{\sin(e^x)} \cdot \frac{d}{dx}\sin(e^x)$$

$$= \frac{1}{\sin(e^x)} \cdot \cos(e^x) \cdot \frac{d}{dx}(e^x)$$

$$= \boxed{\frac{\cos(e^x)\,e^x}{\sin(e^x)}}$$

4. $\exp(\sin(\log(x)))$

$$\frac{d}{dx}\left(e^{\sin(\log(x))}\right)$$

$$= e^{\sin(\log(x))} \cdot \frac{d}{dx}\sin(\log(x))$$

$$= e^{\sin(\log(x))} \cdot \cos(\log(x)) \cdot \frac{d}{dx}\log(x)$$

$$= e^{\sin(\log(x))} \cdot \cos(\log(x)) \cdot \frac{1}{x}$$

$$= \boxed{\frac{e^{\sin(\log(x))} \cdot \cos(\log(x))}{x}}$$

2. $\log(\exp(\sin(x))) = \log(e^{\sin(x)}) = \sin(x)$

$$\frac{d}{dx}(\log(\exp(\sin(x))) = \frac{d}{dx}(\sin(x))$$

$$= \boxed{\cos(x)}$$

3. $\exp(\log(\sin(x)) = \sin(x)$

$$\frac{d}{dx}(\exp(\log(\sin(x))) = \frac{d}{dx}(\sin(x))$$

$$= \boxed{\cos(x)}$$

5. $\sin(\log(\exp(x))) = \sin(\log(e^x)) = \sin(x)$

$$\frac{d}{dx}(\sin(\log(\exp(x))) = \frac{d}{dx}(\sin(x))$$

$$= \boxed{\cos(x)}$$

6. $\sin(\exp(\log(x))) = \sin(e^{\log(x)}) = \sin(x)$

$$\frac{d}{dx}(\sin(e^{\log(x)})) = \frac{d}{dx}(\sin(x))$$

$$= \boxed{\cos(x)}$$

* PROBLEM 32.2

A)

1. $\log(\sin(\exp(x)))$
2. $\log(\exp(\sin(x))) = \sin(x)$ => CAN BE INTEGRATED AS AN ELEMENTARY FUNCTION
3. $\exp(\log(\sin(x))) = \sin(x)$ => CAN BE INTEGRATED AS AN ELEMENTARY FUNCTION
4. $\exp(\sin(\log(x)))$
5. $\sin(\log(\exp(x))) = \sin(x)$ => CAN BE INTEGRATED AS AN ELEMENTARY FUNCTION
6. $\sin(\exp(\log(x))) = \sin(x)$ => CAN BE INTEGRATED AS AN ELEMENTARY FUNCTION

B)

2. $\int \log(\exp(\sin(x)))\,dx$

$\int \sin(x)\,dx$

$= \boxed{-\cos(x) + C}$

3. $\int \exp(\log(\sin))\,dx$

$\int \sin(x)\,dx$

$= \boxed{-\cos(x) + C}$

5. $\int \sin(\log(\exp(x)))\,dx$

$\int \sin(x)\,dx$

$= \boxed{-\cos(x) + C}$

6. $\int \sin(\exp(\log(x)))\,dx$

$\int \sin(x)\,dx$

$= \boxed{-\cos(x) + C}$

* PROBLEM 32.3

$\exp(\sin(x))$

FUNCTIONS THAT CANNOT BE INTEGRATED

1. $\exp(\cos(x))$
2. $\sin(\cos(x))$
3. $\tan(\exp(x))$
4. $\tan(\cos(x))$

* PROBLEM 32.4

APPLIED EXTREMIZATION PROBLEM

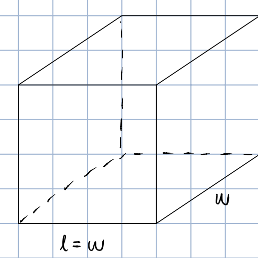
=> WE HAVE $90 m^2$ OF MATERIAL TO BUILD A BOX WITH A SQUARE BASE AND NO TOP. DETERMINE THE DIMENSIONS OF THE BOX THAT WILL MAXIMIZE THE ENCLOSED VOLUME.

$\boxed{l = w}$

$$90 = lw + 2(lh) + 2(wh)$$
$$90 = w^2 + 2wh + 2wh$$
$$90 = w^2 + 4wh$$

$$V = lwh$$
$$V = w^2 h$$

$$h = \frac{90 - w^2}{4w}$$



$$V(w) = w^2\left(\frac{90 - w^2}{4w}\right)$$

$$V(w) = \frac{1}{4} w\left(90 - w^2\right)$$

$$V(w) = \frac{1}{4}\left(90w - w^3\right)$$

$$V'(w) = \frac{1}{4}\left(90 - 3w^2\right) \rightarrow \frac{1}{4}\left(90 - 3w^2\right) = 0$$

$$90 - 3w^2 = 0$$
$$3w^2 = 90$$
$$w^2 = 30$$
$$w = \sqrt{30}$$

$$V''(w) = -\frac{3}{2} w$$

=> SECOND DERIVATIVE OF THIS FUNCTION IS ALWAYS NEGATIVE FOR POSITIVE w

$$l = w = \sqrt{30}$$

$$h = \frac{90 - 30}{4\sqrt{30}}$$

$$\boxed{\begin{gathered} w = \sqrt{30} \\ l = \sqrt{30} \\ h = \frac{90-30}{4\sqrt{30}} \end{gathered}}$$

* PROBLEM 32.5

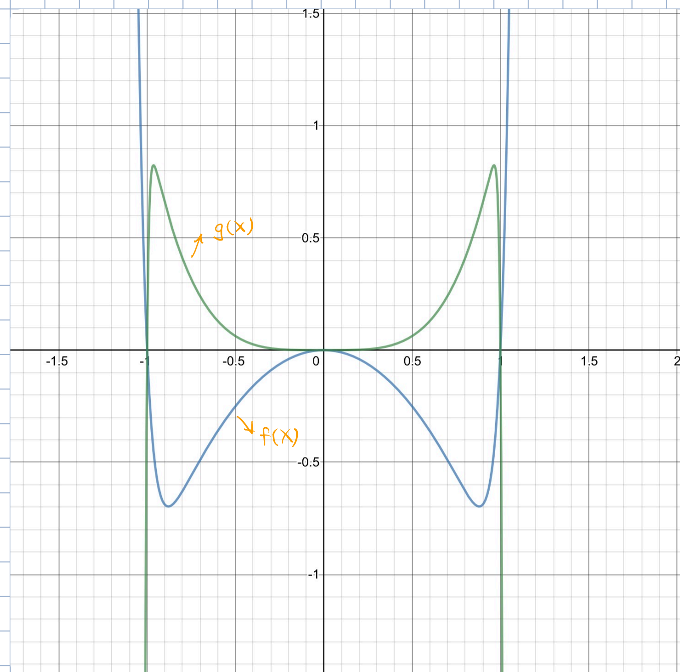
A) AN AREA PROBLEM FOR A REGION IN THE PLANE

=> A REGION IS ENCLOSED BY TWO GRAPHS $f(x) = x^{20} - x^2$ AND $g(x) = x^4 - x^8$. FIND THE AREA.

=> FINDING THE POINTS OF INTERSECTION

$$f(x) = g(x)$$
$$x^{20} - x^2 = x^4 - x^8$$
$$x^{20} + x^8 - x^4 - x^2 = 0$$
$$x^2(x^{18} + x^6 - x^2 - 1) = 0$$

$$x_1 = 0$$
$$x_2 = 1$$
$$x_3 = -1$$



=> CALCULATING THE AREA

$$\int_{-1}^{1} g(x) - f(x) \, dx$$

$$\int_{-1}^{1} x^4 - x^8 - (x^{20} - x^2) \, dx$$

$$\int_{-1}^{1} x^4 - x^8 - x^{20} + x^2 \, dx$$

$$= \frac{x^5}{5} - \frac{x^9}{9} - \frac{x^{21}}{21} + \frac{x^3}{3} \Big|_{-1}^{1}$$

$$= \frac{1}{5} - \frac{1}{9} - \frac{1}{21} + \frac{1}{3} - \left(-\frac{1}{5} + \frac{1}{9} + \frac{1}{21} - \frac{1}{3}\right)$$

$$= \frac{118}{315} + \frac{118}{315}$$

$$= \boxed{\frac{236}{315}}$$

B) A VOLUME PROBLEM FOR A SURFACE OF REVOLUTION

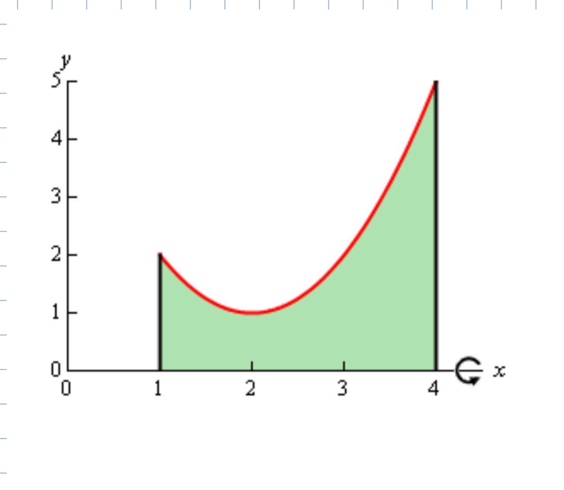
=> DETERMINE THE VOLUME OF THE SOLID OBTAINED BY ROTATING THE REGION BOUNDED BY

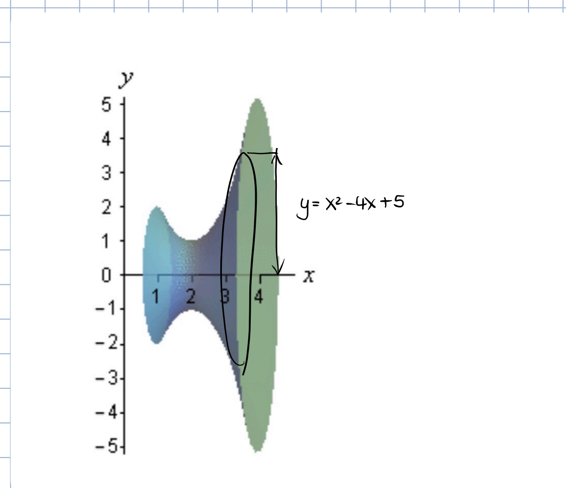
$y = x^2 - 4x + 5$, $x = 1$, $x = 4$

$y = x^2 - 4x + 5$

$x = 1$

$x = 4$





$$A(x) = \pi(x^2 - 4x + 5)^2$$

$$= \pi(x^4 - 8x^3 + 26x^2 - 40x + 25)$$

$$V = \int_a^b A(x)\,dx$$

$$= \int_1^4 \pi(x^4 - 8x^3 + 26x^2 - 40x + 25)$$

$$= \pi \int_1^4 x^4 - 8x^3 + 26x^2 - 40x + 25)$$

$$= \pi\left(\frac{1}{5}x^5 - 2x^4 + \frac{26}{3}x^3 - 20x^2 + 25x\right)\Big|_1^4$$

$$= \boxed{\frac{78\pi}{5}}$$