INTRODUCTION TO CALCULUS

MATH 1A

UNIT 26: WORKSHEET

Problem 1) TF questions (20 points) No justifications are needed.

1) [T] [F] There is a p such that $\int_0^\infty x^{-p}dx$ is finite.

2) [T] [F] The fundamental theorem of calculus assures that $\int_a^b f'(x)\,dx = f(a) - f(b)$.

3) [T] [F] If $\int_0^x f(t)\,dt$ is monotonically increasing in x for $0 \leq x \leq 1$, then $f(x) \geq 0$ on $0 \leq x \leq 1$.

4) [T] [F] For any continuous function f, the integral $\int_a^b f(x)\,dx$ is the area under a curve and therefore always positive or zero.

5) [T] [F] The integral $\int_1^\infty 1/x^p dx$ is convergent for $p > 1$.

6) [T] [F] If $f_c(x)$ has a minimum x_c which is present for $c < 0$ and disappears for $c > 0$, then $c = 0$ is a catastrophe.

7) [T] [F] There are continuous functions for which the anti derivative can not be expressed using known elementary functions.

8) [T] [F] In the movie "gifted", the problem was to integrate an improper integral.

9) [T] [F] An integral $\int_0^1 f(x)\,dx$ for a continuous function $f(x)$ can be approximated arbitrarily well by Riemann sums.

10) [T] [F] If Gabriel's trumpet has base length $1cm$ and we ask the area to be as large as a sheet of paper, then the length of the trumpet does not fit into the observable universe.

Problem 1) Catastrophes (10 points)

Consider the family of functions $f(x) = x^3 + cx$ on the real line.

a) (5 points) Find all critical points of f, depending on c.

b) (2 points) Using the second derivative test, determine which are minima and which are maxima.

b) (3 points) For which value of c does a catastrophe occur?

Problem 2) Integrals (10 points)

Which method do we use? (Use either substitution, integration by parts or memory where you now the integral).

a) $\int (x-3)\sin(5x)\ dx$

b) $\int \log(5x)\ dx$

c) $\int \frac{e^x}{1+e^{2x}}\ dx$

d) $\int (x-1)^4 \sin(3x)\ dx$

e) $\int \frac{1}{\sqrt{1-x^2}}\ dx$