

# INTRODUCTION TO CALCULUS

MATH 1A

## UNIT 35: WORKSHEET

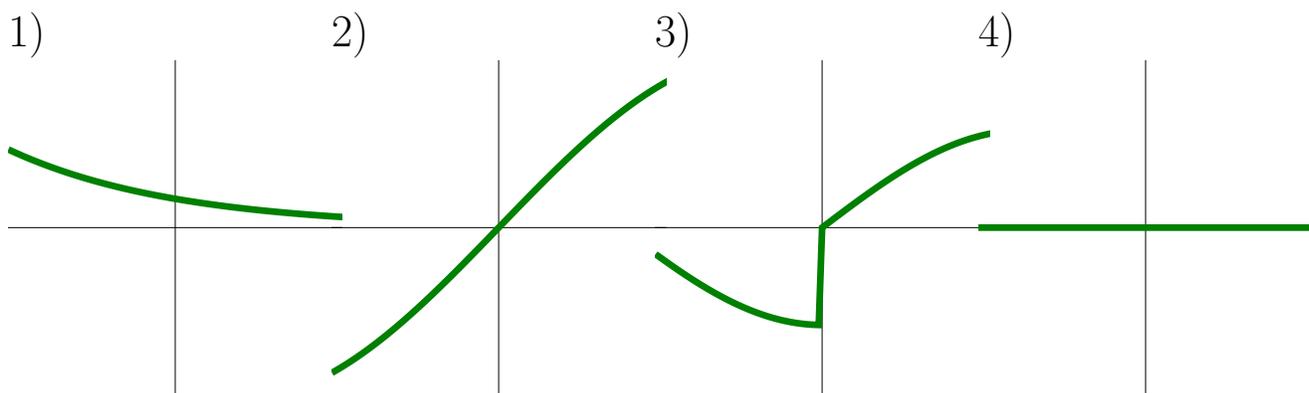
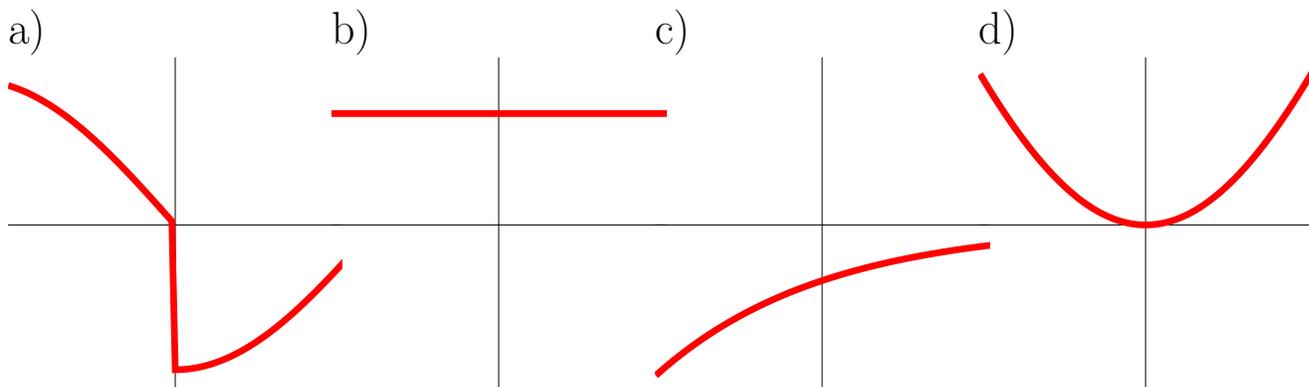
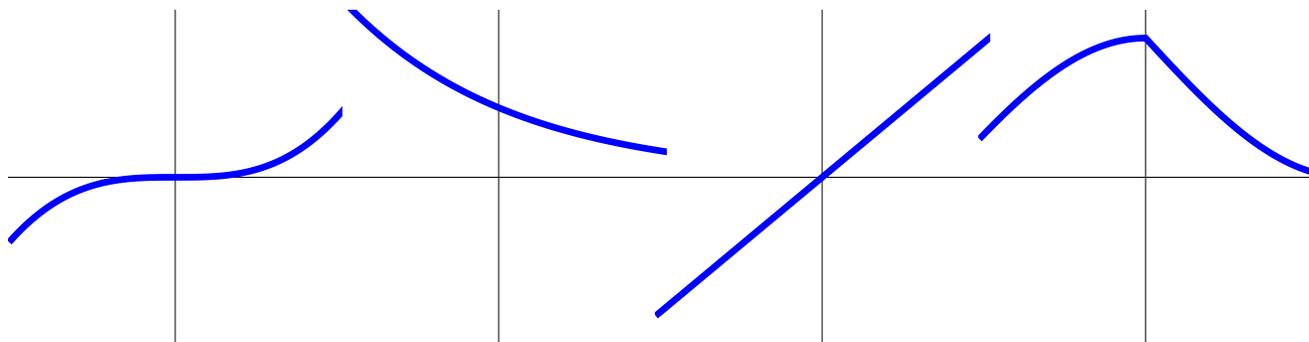
**1:** Name dropping: Match results with names

Result	Enter A-G
Fundamental theorem of trigonometry	
Newton stpdf	
Fundamental theorem of calculus	
Mean value theorem	
Rolle's theorem	
Intermediate value theorem	
Fermat theorem	

A)	$\int_0^1 f'(x) dx = f(1) - f(0)$
B)	$\lim_{x \rightarrow 0} \sin(x)/x = 1$
C)	$f(0) = -1, f(1) = 1$ implies $f(x) = 0$ for some $x \in (0, 1)$ .
D)	$f$ is continuous on $[0, 1]$ then $f$ has a global max and min on $[0, 1]$ .
E)	$T(x) = x - f(x)/f'(x)$ .
F)	If $f(0) = f(1) = 0$ then $f'(x) = 0$ for some $x \in (0, 1)$ .
G)	There exists $x$ in $(0, 1)$ such that $f'(x) = f(1) - f(0)$ .

**2:** Match the following functions (a-d) with their derivatives (1-4) and second derivatives (A-D).

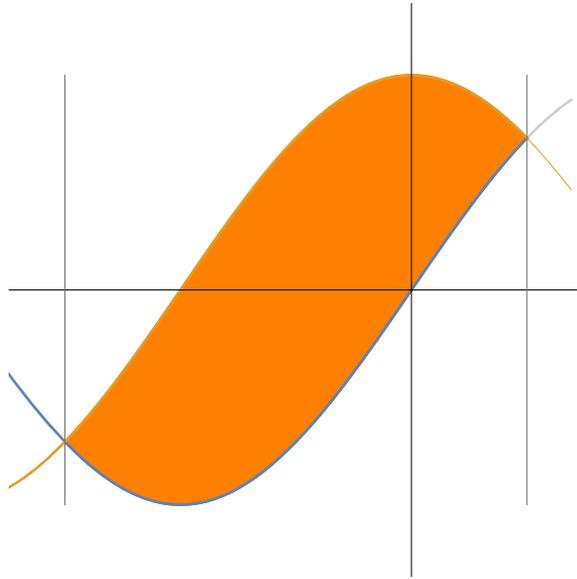
Function a)-d)	Fill in 1)-4)	Fill in A)-D)
graph a)		
graph b)		
graph c)		
graph d)		



A)                      B)                      C)                      D)

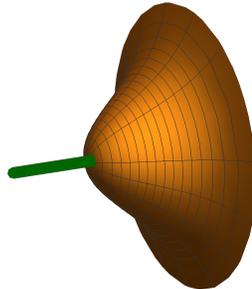
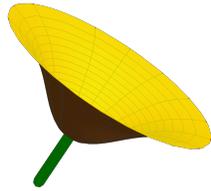
**3:**

Find the area enclosed by the functions  $f(x) = \sin(x)$  and  $f(x) = \cos(x)$  and between the vertical lines  $x = -3\pi/4$  and  $x = \pi/4$ .



**4:**

If we rotate the graph of the function  $f(x) = \sqrt{\tan(x)}$  from  $x = 0$  to  $x = \pi/4$  we obtain a flower shaped solid. The two pictures below allow to admire it from two sides. Find its volume.



**5:**

a) Find the integral or state that it does not exist

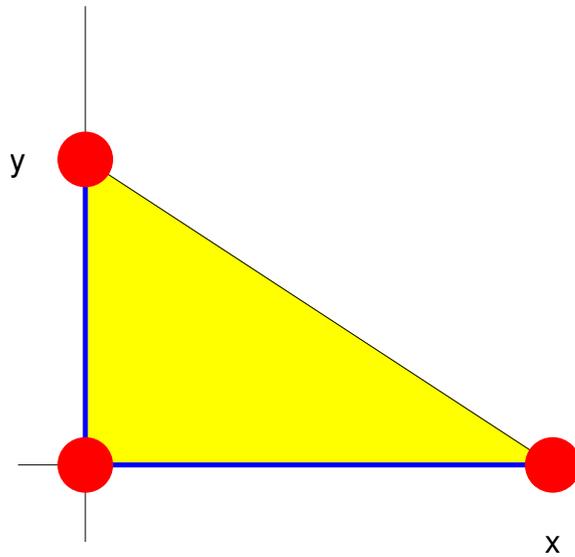
$$\int_1^{\infty} \frac{1}{x^5} dx .$$

b) Find the integral or state that it does not exist

$$\int_0^{\infty} \frac{1}{\cos^2(x)} dx .$$

**6:**

Find the local and global maxima of  $f(x) = 2x^3 - 3x^2$  on the interval  $[-1, 2]$ . Use the second derivative test to check local extrema.



**7:**

Find the anti-derivative:

$$\int x^4 \sin(x - 1) dx .$$

**8:**

- a) Solve the integral  $\int \sin^4(x) \cos(x) dx$ .
- b) Solve the integral  $\int \sqrt{1 + 2x} dx$ .

**9:**  
Find

$$\int_4^5 \frac{1}{(x-2)(x-3)} dx .$$

**10:**

Find the anti-derivatives of the following functions:

a)  $f(x) = \log(x)/x$ .

b)  $f(x) = \frac{1}{x^2-4}$ .

c)  $f(x) = \frac{1}{x \log(x)}$ .

**11:**

If  $F(x) = \log(x)$  is the total cost and  $f(x) = F'(x)$  is the marginal cost and  $g(x) = \log(x)/x$  is the average cost:

- a) Find the break-even point  $f = g$ .
- b) Where is the average cost maximal?
- c) Sweet surprise: why are the results in a) b) the same? The answer to c) is one word.