Math 1A: Introduction to functions and calculus Oliver Knill, Spring 2020

3/4/2020: First hourly

Your Name:

- Start by writing your name in the above box.
- Try to answer each question on the same page as the question is asked. If needed, use the back or the next empty page for work.
- Do not detach pages from this exam packet or unstaple the packet.
- Please write neatly. Answers which are illegible for the grader can not be given credit.
- Except for multiple choice problems, give computations.
- No notes, books, calculators, computers, or other electronic aids are allowed.
- You have 75 minutes time to complete your work.

1		20
2		10
3		10
4		10
5		10
6		10
7		10
8		10
9		10
Total:		100

Problem 1) TF questions (20 points) No justifications are needed.

1) [T] [F] The function $f(x) = \log(x)$ has a root at $x = 1$.

2) [T] [F] $f(x) = x^2 + \exp(-x^2)$ has a root on the real axes.

3) [T] [F] $\cos(\pi) = -1$.

4) [T] [F] $f(x) = \sin(|x|)$ is differentiable everywhere.

5) [T] [F] The chain rule assures that $\frac{d}{dx}\sin(x^5) = \cos(x)5x^4$.

6) [T] [F] The function $f(x) = \exp(-\sin(x))$ is continuous everywhere.

7) [T] [F] $\sinh(x) = (e^x - e^{-x})/2$ is positive everywhere.

8) [T] [F] $\cot(x)$ is the inverse function of $\tan(x)$.

9) [T] [F] If $f(x)$ is differentiable at 0, then $f(x)^2$ is differentiable at 0.

10) [T] [F] The Newton step for a function f is $T(x) = x - f(x)/f'(x)$.

11) [T] [F] $\sin(3\pi/4) = -1$.

12) [T] [F] $f(x) = \tan(x)$ has a vertical asymptote at $x = 0$.

13) [T] [F] e^{x^2} takes the value π at some point.

14) [T] [F] If $f(x) = x^2$, then $Df(x) = f(x+1) - f(x) = 2x + 1$.

15) [T] [F] The intermediate value theorem implies Rolles theorem.

16) [T] [F] The function $\sin(1/x)$ is continuous everywhere.

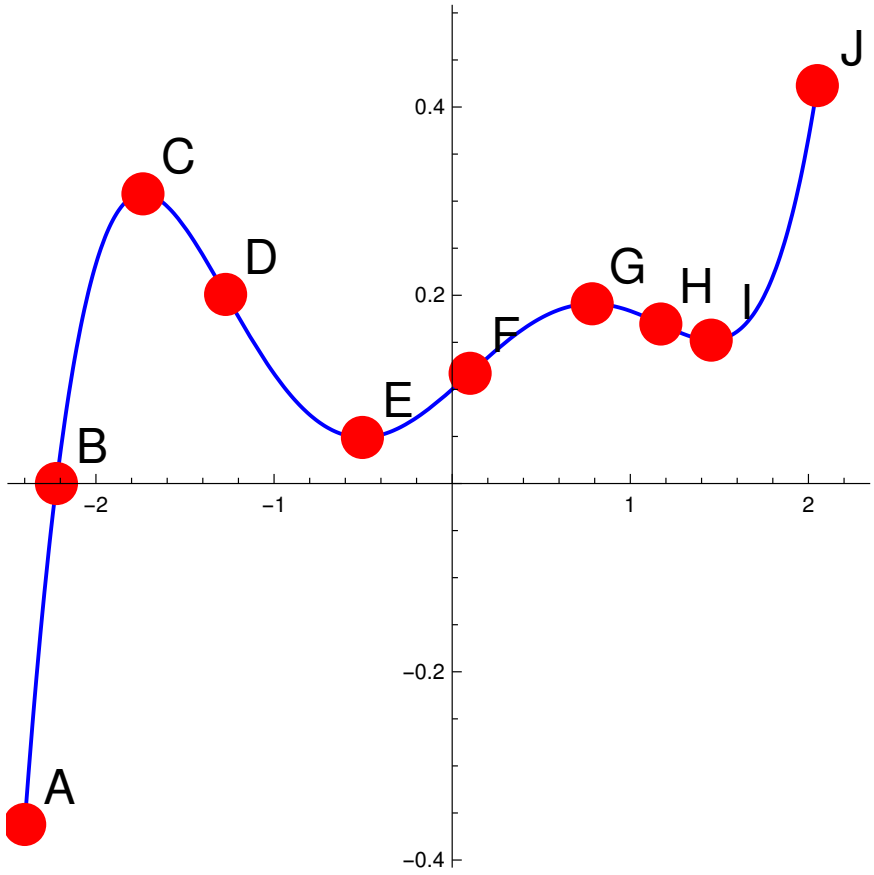
17) [T] [F] $\frac{d}{dx}\arctan(x) = 1/(1+x^2)$.

18) [T] [F] $\frac{d}{dx}\log(3+x) = 3/(3+x)$.

19) [T] [F] A continuous function on $[0, 1)$ has at least one maximum.

20) [T] [F] The derivative of f/g is $(fg' - f'g)/g^2$.

Problem 2) Choice problem (10 points) No justifications are needed.



a) (2 points) List the points A-J which are roots of f.

b) (2 points) List the points A-J are inflection points.

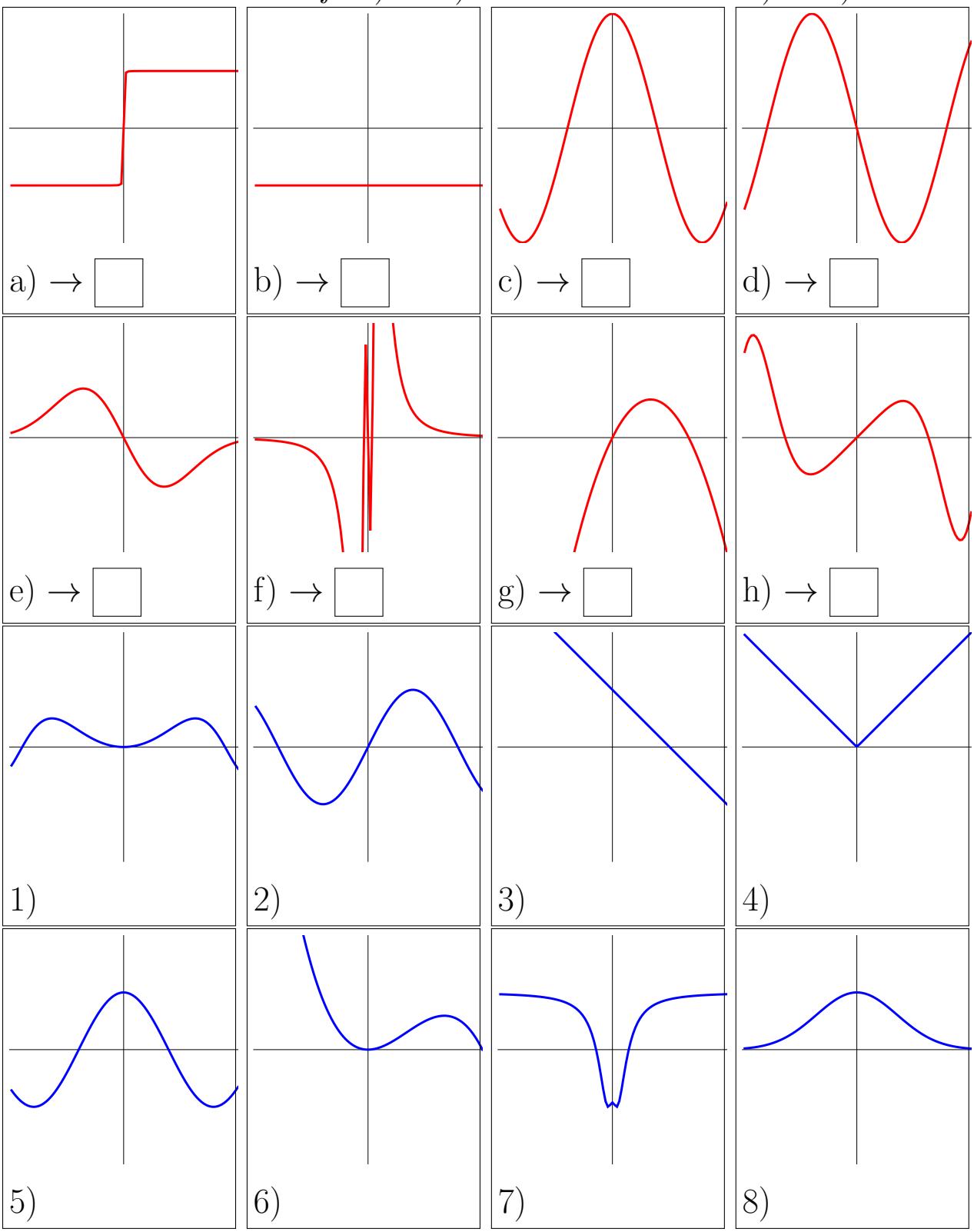
c) (2 points) List the points A-J that are local maxima.

d) (2 points) List the points A-J that are local minima.

e) (2 points) List the points A-J that are global maxima.

Problem 3) Matching problem (10 points) No justifications are needed.

Match the derivatives f' a) to h) with the functions 1) to 8).



Problem 4) Continuity (10 points)

Which of the following functions are continuous on $[-1, 1]$? As usual we extend the domain of definition to points, where a continuation is possible. In each case make the decision "continuous" or "not continuous" and point to the x value which needs special attention.

a) (2 points) $f(x) = \frac{x^6-1}{x^2-1}$

b) (2 points) $f(x) = \frac{\sin(\sin(x))}{\sin(\sin(\sin(x)))}$.

c) (2 points) $f(x) = \frac{\sin^2(x)}{2+\sin(x^2)}$

d) (2 points) $f(x) = \log|x|e^x$

e) (2 points) $f(x) = \frac{\sin(\tan(x))}{\sin(x)}$

Problem 5) Derivatives (10 points)

Find the derivatives of the following functions. In each case, in-

dicate which differentiation rule you use.

a) (2 points) $f(x) = \frac{1}{1+e^x}$.

b) (2 points) $f(x) = \cos(x)\sin(x)$.

c) (2 points) $f(x) = \frac{1+x^3}{1+x^2}$.

d) (2 points) $f(x) = \arctan(\sin(x))$.

e) (2 points) $f(x) = \log(\log(5x))$.

Problem 6) Limits (10 points)

Find the limits $\lim_{x\to 0} f(x)$ for the following functions:

a) (2 points) $f(x) = \frac{\exp(7x)-1}{\exp(4x)-1}$.

b) (2 points) $f(x) = \frac{x-1}{x+1}$.

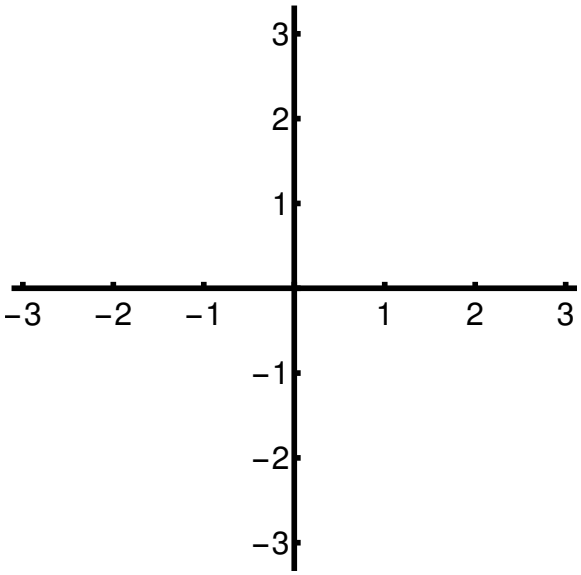
c) (2 points) $f(x) = \frac{\arctan(x)}{\sin(x)}$.

d) (2 points) $f(x) = \frac{\log(x^3)}{\log(x^2)}$.

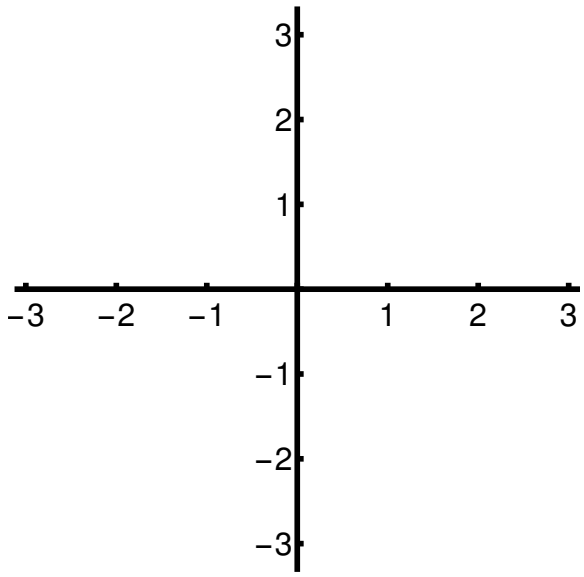
e) (2 points) $f(x) = \frac{\sin(3x)\sin(5x)}{\sin(7x)\sin(2x)}$.

Problem 7) Functions (10 points)

a) (5 points) Draw the graph of the natural log function $f(x) = \log|x| = \ln|x|$. Make sure you get the roots and asymptotes correct.



b) (5 points) Draw the graph of the arctan function $f(x) = \arctan(x)$. Make sure you get the roots and asymptotes correct.



Problem 8) Extrema (10 points)

a) (3 points) Find all the critical points of the function

$$f(x) = x^3 - 3x + 1 \ .$$

b) (3 points) Use the second derivative test to classify the critical points of f.

c) (2 points) On the interval $[-3, 3]$, where it the global maximum, and where is the global minimum?

d) (2 points) Which theorem assures that there is a global maximum and a global minimum on $[-3, 3]$?

Problem 9) Algebra rules (10 points, 2 points each)

a)	$(e^x)^y$	
b)	e^{x+y}	
c)	$\log(xy)$	
d)	$\frac{\tan(x)}{\sin(x)}$	
e)	$\frac{x^9}{x^3}$	

Choose from the following expressions.

- $1/\cos(x)$
- $\cos(x)/\sin^2(x)$.
- $e^x e^y$
- $\log(x+y)$
- $e^x + e^y$
- $e^{(xy)}$
- x^6
- $\log(x) - \log(y)$
- $e^{(x^y)}$
- $\log(x) + \log(y)$
- x^3
- $\cos(x)$