

3/4/2020: First hourly

Your Name:

- Start by writing your name in the above box.
- Try to answer each question on the same page as the question is asked. If needed, use the back or the next empty page for work.
- Do not detach pages from this exam packet or unstaple the packet.
- Please write neatly. Answers which are illegible for the grader can not be given credit.
- Except for multiple choice problems, give computations.
- No notes, books, calculators, computers, or other electronic aids are allowed.
- You have 75 minutes time to complete your work.

1		20
2		10
3		10
4		10
5		10
6		10
7		10
8		10
9		10
Total:		100

Problem 1) TF questions (20 points) No justifications are needed.

- 1) T F The function $f(x) = \log(x)$ has a root at $x = 1$.

Solution:

Yes, $\log(1)=0$.

- 2) T F $f(x) = x^2 + \exp(-x^2)$ has a root on the real axes.

Solution:

No, it is always positive

- 3) T F $\cos(\pi) = -1$.

Solution:

You know that.

- 4) T F $f(x) = \sin(|x|)$ is differentiable everywhere.

Solution:

It is not at $x = 0$ or $x = k\pi$ with integer k .

- 5) T F The chain rule assures that $\frac{d}{dx} \sin(x^5) = \cos(x)5x^4$.

Solution:

This is not the correct application.

- 6) T F The function $f(x) = \exp(-\sin(x))$ is continuous everywhere.

Solution:

It is a combination of continuous functions

- 7) T F $\sinh(x) = (e^x - e^{-x})/2$ is positive everywhere.

Solution:

It is zero at $x = 0$ and negative for $x < 0$.

- 8) T F $\cot(x)$ is the inverse function of $\tan(x)$.

Solution:

It is the arctan, not the cotan

- 9) T F If $f(x)$ is differentiable at 0, then $f(x)^2$ is differentiable at 0.

Solution:

Differentiate to get $2f(x)f'(x)$.

- 10) T F The Newton step for a function f is $T(x) = x - f(x)/f'(x)$.

Solution:

yes, this is the case

- 11) T F $\sin(3\pi/4) = -1$.

Solution:

It is positive and $\sqrt{1/2}$

- 12) T F $f(x) = \tan(x)$ has a vertical asymptote at $x = 0$.

Solution:

The cot function has this property.

- 13) T F e^{x^2} takes the value π at some point.

Solution:

Yes, by the intermediate value theorem, there is a place, where the value is 1 and places where f is arbitrary large

- 14) T F If $f(x) = x^2$, then $Df(x) = f(x + 1) - f(x) = 2x + 1$.

Solution:

Yes, one check this formally $(x + 1)^2 - x^2 = 2x + 1$.

- 15) T F The intermediate value theorem implies Rolles theorem.

Solution:

It is the mean value theorem which implies this.

- 16) T F The function $\sin(1/x)$ is continuous everywhere.

Solution:

It is a basic oscillatory discontinuity.

- 17) T F $\frac{d}{dx} \arctan(x) = 1/(1 + x^2)$.

Solution:

An important one.

18) T F $\frac{d}{dx} \log(3 + x) = 3/(3 + x).$

Solution:

It is $1/(3 + x).$

19) T F A continuous function on $[0, 1)$ has at least one maximum.

Solution:

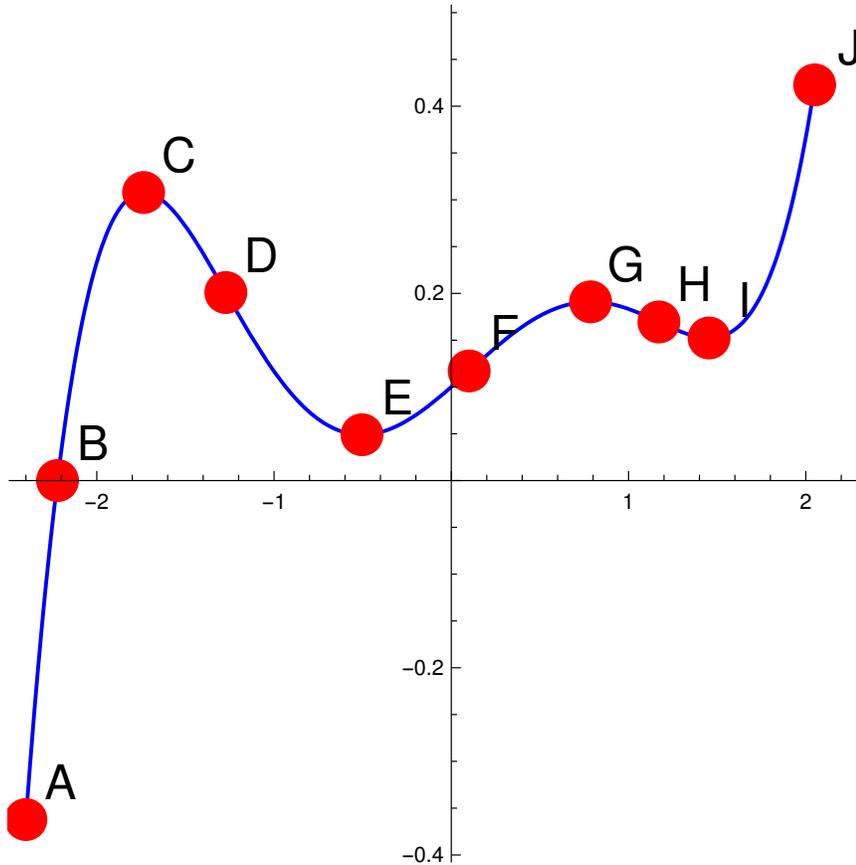
The interval is not closed. Bolzano does not apply.

20) T F The derivative of f/g is $(fg' - f'g)/g^2.$

Solution:

It is low D high minus high d low!

Problem 2) Choice problem (10 points) No justifications are needed.



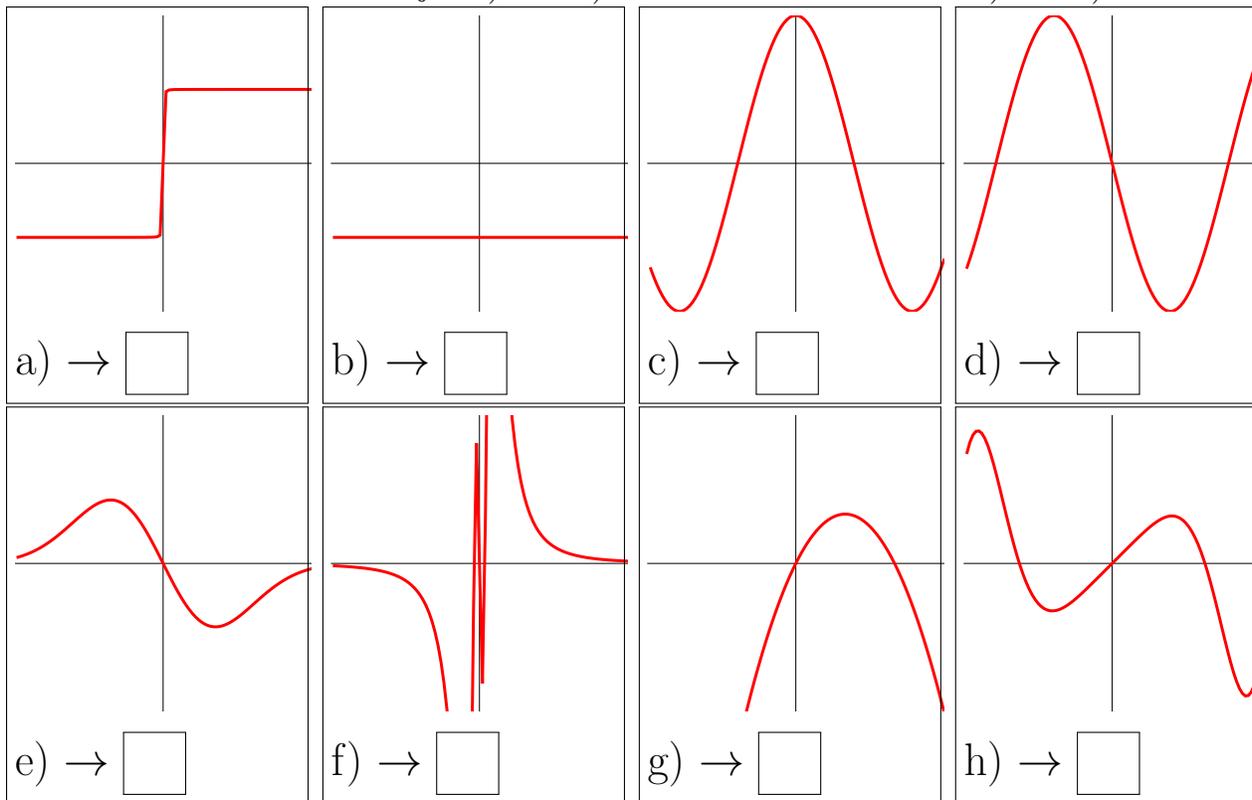
- (2 points) List the points A-J which are roots of f .
- (2 points) List the points A-J are inflection points.
- (2 points) List the points A-J that are local maxima.
- (2 points) List the points A-J that are local minima.
- (2 points) List the points A-J that are global maxima.

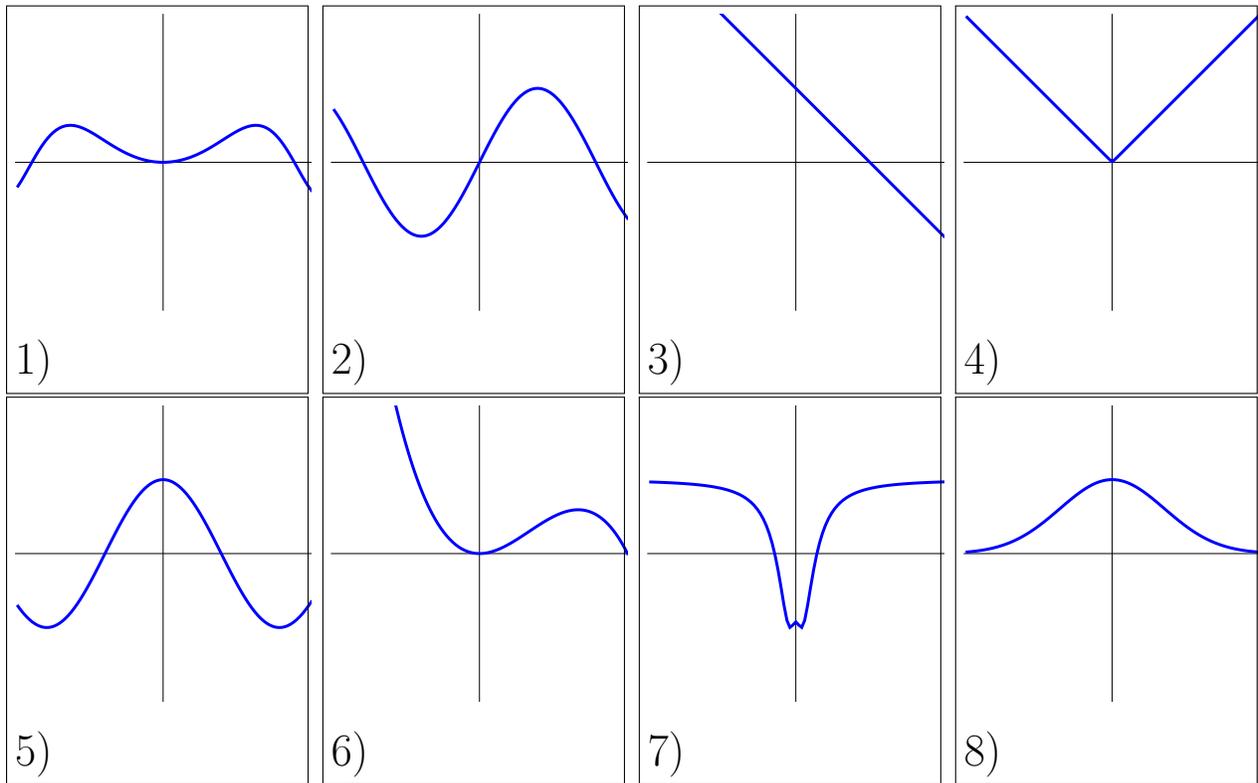
Solution:

- a) B
- b) D, F, H .
- c) C, G
- d) E, I
- e) J .

Problem 3) Matching problem (10 points) No justifications are needed.

Match the derivatives f' a) to h) with the functions 1) to 8).





Solution:

4325

8761

Problem 4) Continuity (10 points)

Which of the following functions are continuous on $[-1, 1]$? As usual we extend the domain of definition to points, where a continuation is possible. In each case make the decision “continuous” or “not continuous” and point to the x value which needs special attention.

a) (2 points) $f(x) = \frac{x^6-1}{x^2-1}$

b) (2 points) $f(x) = \frac{\sin(\sin(x))}{\sin(\sin(\sin(x)))}$.

c) (2 points) $f(x) = \frac{\sin^2(x)}{2+\sin(x^2)}$

d) (2 points) $f(x) = \log|x|e^x$

e) (2 points) $f(x) = \frac{\sin(\tan(x))}{\sin(x)}$

Solution:

a) Continuous. Hospital gives the limit $6/2$ at $x = 0$.

b) Continuous. There is only point 0 which needs to be considered. The limit at $x = 0$ is 1.

c) Continuous. The denominator is always positive. The function is continuous.

d) Not continuous. There is no way to save this at $x = 0$ as $\log|x|$ goes to $-\infty$ and $e^0 = 1$.

e) Continuous. We can use Hospital to get the value 1 at $x = 0$ or $x = \pi$ etc.

Problem 5) Derivatives (10 points)

Find the derivatives of the following functions. In each case, indicate which differentiation rule you use.

a) (2 points) $f(x) = \frac{1}{1+e^x}$.

b) (2 points) $f(x) = \cos(x) \sin(x)$.

c) (2 points) $f(x) = \frac{1+x^3}{1+x^2}$.

d) (2 points) $f(x) = \arctan(\sin(x))$.

e) (2 points) $f(x) = \log(\log(5x))$.

Solution:

a) $e^x/(1 + e^x)$.

b) $\cos^2(x) - \sin^2(x)$.

c) $[3x^2(1 + x^2) - (1 + x^3)2x]/(1 + x^2)^2$

d) $\cos(x)/(1 + \sin^2(x))$. e) $1/(x \log(5x))$.

Problem 6) Limits (10 points)

Find the limits $\lim_{x \rightarrow 0} f(x)$ for the following functions:

a) (2 points) $f(x) = \frac{\exp(7x)-1}{\exp(4x)-1}$.

b) (2 points) $f(x) = \frac{x-1}{x+1}$.

c) (2 points) $f(x) = \frac{\arctan(x)}{\sin(x)}$.

d) (2 points) $f(x) = \frac{\log(x^3)}{\log(x^2)}$.

e) (2 points) $f(x) = \frac{\sin(3x)\sin(5x)}{\sin(7x)\sin(2x)}$.

Solution:

All with Hospital.

a) $7/4$.

b) -1 (no limit needs to be taken as the values are finite).

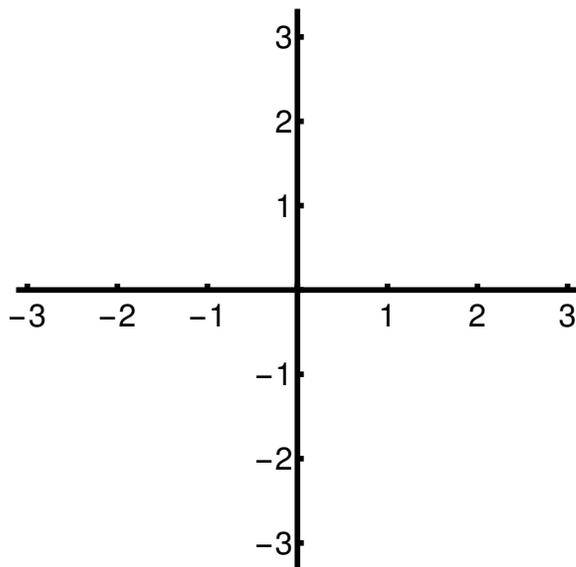
c) 1

d) $3/2$

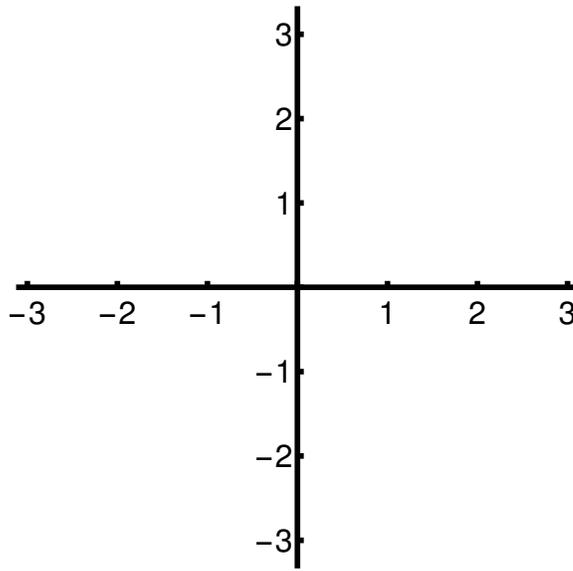
e) Take apart: $\sin(3x)/\sin(7x)$ goes to $3/7$ and $\sin(5x)/\sin(2x)$ goes to $5/2$. We have $15/14$.

Problem 7) Functions (10 points)

a) (5 points) Draw the graph of the natural log function $f(x) = \log|x| = \ln|x|$. Make sure you get the roots and asymptotes correct.



b) (5 points) Draw the graph of the arctan function $f(x) = \arctan(x)$. Make sure you get the roots and asymptotes correct.



Problem 8) Extrema (10 points)

a) (3 points) Find all the critical points of the function

$$f(x) = x^3 - 3x + 1 .$$

b) (3 points) Use the second derivative test to classify the critical points of f .

c) (2 points) On the interval $[-3, 3]$, where is the global maximum, and where is the global minimum?

d) (2 points) Which theorem assures that there is a global maximum and a global minimum on $[-3, 3]$?

Solution:

- a) We have $f'(x) = 3x^2 - 3$ which is zero at $x = 1$ or $x = -1$.
- b) The second derivative is $6x$. So $x = 1$ is a minimum and $x = -1$ is a maximum.
- c) The global minimum is $x = 3$ and the minimum is $x = -3$.
- d) The Bolzano extremal value theorem.

Problem 9) Algebra rules (10 points, 2 points each)

a)	$(e^x)^y$	
b)	e^{x+y}	
c)	$\log(xy)$	
d)	$\frac{\tan(x)}{\sin(x)}$	
e)	$\frac{x^9}{x^3}$	

Choose from the following expressions.

- $1/\cos(x)$
- $\cos(x)/\sin^2(x)$.
- $e^x e^y$
- $\log(x + y)$
- $e^x + e^y$
- $e^{(xy)}$
- x^6
- $\log(x) - \log(y)$

- $e^{(x^y)}$
- $\log(x) + \log(y)$
- x^3
- $\cos(x)$

Solution:

a) e^{xy}

b) $e^x e^y$.

c) $\log(x) + \log(y)$ d) $1/\cos(x)$.

e) x^6 .