

4/8/2020: First hourly Practice D

Your Name:

- Solutions are submitted as PDF handwritten in a file called after your name. Capitalize the first letters like OliverKnill.pdf. Hand-write your paper. It can be handwritten also electronically using an ipad but it needs to **feature your personal handwriting** and contain no typed part. If you like, you can start writing on a new paper. For 1), you could write 1: False, 2: False \cdots 20: False. Also, please sign your solutions.
- No books, calculators, computers, or other electronic aids are allowed. You can use one page of your own handwritten notes when writing the paper.
- The exam will be released at 9 AM on April 8. Try to do it during regular class time. You have to submit the paper within 24 hours by emailing it to knill@math.harvard.edu as an attachment. Submit it soon so that technical problems can be ironed out. It is your responsibility to submit the paper on time and get within that time also a confirmation.

1		20
2		10
3		10
4		10
5		10
6		10
7		10
8		10
9		10
Total:		100

Problem 1) TF questions (20 points) No justifications are needed.

- 1) T F The formula $\int_0^x f''(x) dx = f'(x) - f'(0)$ holds.
- 2) T F The area of the upper half disc is the integral $\int_{-1}^1 \sqrt{1-x^2} dx$
- 3) T F If the graph of the function $f(x) = x^2$ is rotated around the interval $[0, 1]$ in the x axes we obtain a solid with volume $\int_0^1 \pi x^4 dx$.
- 4) T F The function $f(x) = e^x$ is the only anti derivative of e^x .
- 5) T F If f has a critical point at 1, then $F(x) = \int_0^x f(t) dt$ has an inflection point at 1.
- 6) T F Catastrophes are parameter values c for a family of functions $f_c(x)$, for which a local minimum of f_c disappears.
- 7) T F The volume of a cylinder of height and radius 1 minus the volume of a cone of height and radius 1 is half the volume of a sphere of radius 1.
- 8) T F Rolle's theorem tells that if $0 < c < 1$ is a critical point of f and $f(0) = f(1)$, then the critical point is in the interval $[0, 1]$.
- 9) T F Rolle also introduced the notation $|x|^{1/3}$ for roots.
- 10) T F Integrals are linear: $\int_0^x f(t) + g(t) dt = \int_0^x f(t) dt + \int_0^x g(t) dt$.
- 11) T F The function $\text{Li}(x) = \int_2^x dt/\log(t)$ has an anti-derivative which is a finite construct of trig functions.
- 12) T F There is a region enclosed by the graphs of x^5 and x^6 which is finite and positive.
- 13) T F The integral $\int_{-1}^1 1/x^4 dx = -1/(5x^5)|_{-1}^1 = -1/5 - 1/5 = -2/5$ is defined and negative.
- 14) T F Gabriel's trumpet has finite volume but infinite surface area.
- 15) T F A function $f(x)$ is a probability density function, if $f(x) \geq 0$ and $\int_{-\infty}^{\infty} f(x) dx = 1$.
- 16) T F The mean of a function on an interval $[a, b]$ is $\int_a^b f(x) dx$.
- 17) T F The cumulative probability density function is an antiderivative of the probability density function.
- 18) T F The integral $\int_{-\infty}^{\infty} (x^2 - 1) dx$ is finite.
- 19) T F The total prize is the derivative of the marginal prize.
- 20) T F The acceleration is the anti-derivative of the velocity.

Problem 2) Matching problem (10 points) No justifications are needed.

Match the following functions with their anti derivatives. Of course only 6 of the 30 functions will appear.

Function	Antiderivative Enter 1-30
$\cos(3x)$	
$\sin(3x)$	
$3x$	

Function	Antiderivative Enter 1-30
$1/(3x)$	
$\tan(3x)$	
$1/(1 + 9x^2)$	

- | | | |
|------------------|---------------------|-------------------------|
| 1) $\sin(3x)$ | 6) $\cos(3x)$ | 11) $\log(x)/3$ |
| 2) $-\sin(3x)/3$ | 7) $-\cos(3x)/3$ | 12) $1/(3-x)$ |
| 3) $\sin(3x)/3$ | 8) $\cos(3x)/3$ | 13) $1/(3x)$ |
| 4) $-3\sin(3x)$ | 9) $-3\cos(3x)$ | 14) $\log(x/3)$ |
| 5) $3\sin(3x)$ | 10) $3\cos(3x)$ | 15) $-1/(3x^2)$ |
| 16) $3x^2$ | 21) $\arctan(3x)/3$ | 26) $1/\cos^2(3x)$ |
| 17) $x^2/2$ | 22) $3\arctan(3x)$ | 27) $\log(\cos(3x))$ |
| 18) $3x^2/2$ | 23) $1/(1+9x^2)$ | 28) $-\log(\cos(3x))/3$ |
| 19) 3 | 24) $3/(1+9x^2)$ | 29) $\log(\cos(3x))/3$ |
| 20) x^2 | 25) $-3/(1+x^2)$ | 30) $3/\cos^3(3x)$ |

Problem 3) Matching problem (10 points) No justifications are needed.

Which of the following formulations is a Riemann sum approximating the integral $\int_0^3 f(x) dx$ of $f(x) = x^2$ over the interval $0, 3$.

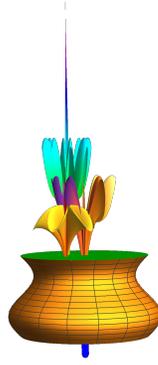
Sum	Check if this is the Riemann sum.
$n \sum_{k=0}^{n-1} (3k/n)^2$	
$\frac{1}{n} \sum_{k=0}^{n-1} (3k/n)^2$	
$n \sum_{k=0}^{3n-1} (k/n)^2$	
$\frac{1}{n} \sum_{k=0}^{3n-1} (k/n)^2$	

Problem 4) Area computation (10 points)

Find the area of the region enclosed by the three curves $y = 6 - x^2$, $y = -x$ and $y = x$ which is above the x axes.

Problem 5) Volume computation (10 points)

Emma R. grows magical plants in a pot which is a rotationally symmetric solid for which the radius at position x is $5 + \sin(x)$ and $0 \leq x \leq 2\pi$. Find the volume of the pot.



Problem 6) Definite integrals (10 points)

Find the following definite integrals

a) (5 points) $\int_1^2 x^{1/5} + x^4 + 1/x \, dx$.

b) (5 points) $\int_1^3 2x + \sin(x - 1) + \cos(x + 2) \, dx$

Problem 7) Anti-derivatives (10 points)

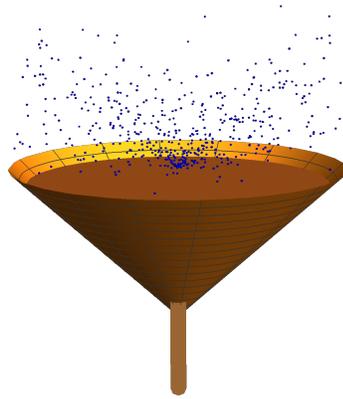
Find the following anti-derivatives

a) (5 points) $\int \frac{3}{\sqrt{1-x^2}} + x^4 + \frac{1}{1+x^2} \, dx$

b) (5 points) $\int \frac{1}{x-2} + \frac{1}{x+4} + \frac{2}{x-1} \, dx$

Problem 8) Related rates (10 points)

A coffee machine has a filter which is a cone of radius z at height z . Coffee spills out at a rate of 1 cubic centimeter per second. How fast does the water level sink at height $z = 10$?



Problem 9) PDF's and CDF's (10 points)

Assume we know that the CDF is given by $f(x) = 1/2 + \arctan(x)/\pi$.
Determine the corresponding PDF.

Problem 10) Improper integrals (10 points)

Evaluate the following improper integrals or state that they do not exist

a) (3 points) $\int_1^{\infty} 1/\sqrt{x} dx$.

b) (2 points) $\int_0^1 \sqrt{x} dx$.

c) (3 points) $\int_0^{\infty} 2xe^{-x^2} dx$.

d) (2 points) $\int_0^{\infty} \frac{1}{x} dx$