

4/8/2020: First hourly Practice E

Your Name:

- Solutions are submitted as PDF handwritten in a file called after your name. Capitalize the first letters like OliverKnill.pdf. Hand-write your paper. It can be handwritten also electronically using an ipad but it needs to **feature your personal handwriting** and contain no typed part. If you like, you can start writing on a new paper. For 1), you could write 1: False, 2: False \cdots 20: False. Also, please sign your solutions.
- No books, calculators, computers, or other electronic aids are allowed. You can use one page of your own handwritten notes when writing the paper.
- The exam will be released at 9 AM on April 8. Try to do it during regular class time. You have to submit the paper within 24 hours by emailing it to knill@math.harvard.edu as an attachment. Submit it soon so that technical problems can be ironed out. It is your responsibility to submit the paper on time and get within that time also a confirmation.

1		20
2		10
3		10
4		10
5		10
6		10
7		10
8		10
9		10
Total:		100

Problem 1) TF questions (20 points) No justifications are needed.

- 1) T F The formula $\int_0^x f''(x) dx = f'(x) - f'(0)$ holds.
- 2) T F The area of the lower half disc is the integral $\int_{-1}^1 -\sqrt{1-x^2} dx$
- 3) T F If the graph of the function $f(x) = x^2$ is rotated around the interval $[0, 1]$ we obtain a solid with volume $\int_0^1 \pi x^4 dx$.
- 4) T F The identity $d/dx \int_0^x f''(t) dt = f'(x)$ holds.
- 5) T F There is a point in $[0, 1]$, where $f'(x) = 0$ if $f(x) = x^3 - x^2 + 1$.
- 6) T F The fundamental theorem of calculus assures that $\int_a^b f'(x) dx = f(b) - f(a)$.
- 7) T F If f is differentiable on $[a, b]$, then $\int_a^b f(x) dx$ exists.
- 8) T F The integral $\int_0^{\pi/2} \sin(\sin(x)) dx$ is positive.
- 9) T F The anti-derivative of an anti-derivative of f is equal to the derivative of f .
- 10) T F If a function is positive everywhere, then $\int_a^b f(x) dx$ is positive too.
- 11) T F If a differentiable function is odd, then $\int_{-1}^1 f(x) dx = 0$.
- 12) T F If $f_c(x)$ is a function with a local minimum at 0 for all $c < 0$ and no local minimum in $[-1, 1]$ for $c > 0$, then $c = 0$ is called a catastrophe.
- 13) T F The term "improper integral" is a synonym for "indefinite integral".
- 14) T F The function $F(x) = x \sin(x)$ is an antiderivative of $\sin(x)$.
- 15) T F The mean value theorem holds for every continuous function.
- 16) T F Newton and Leibniz were best buddies all their life. Leibniz even gave once the following famous speech: "You guys might not know this, but I consider myself a bit of a loner. I tend to think of myself as a one-man wolf pack. But when my sister brought Isaac home, I knew he was one of my own. And my wolf pack ... it grew by one.
- 17) T F Any function $f(x)$ satisfying $f(x) > 0$ is a probability density function.
- 18) T F The moment of inertia integral I can be used to compute energy with the relation $E = \omega^2 I / 2$ where ω is the angular velocity.
- 19) T F If $0 \leq f(x) \leq g(x)$ then $0 \leq \int_0^1 f(x) dx \leq \int_0^1 g(x) dx$.
- 20) T F The improper integral $\int_0^\infty 1/(x^4 + 1) dx$ is finite.

Problem 2) Matching problem (10 points) No justifications are needed.

From the following functions there are two for which no elementary integral is found. Find them. You can find them by spotting the complement set of functions which you can integrate.

Function	Antiderivative is not elementary	Function	Antiderivative is not elementary
e^{-x^2}		$1/\log(x)$	
$\sin(3x)$		$\tan(3x)$	
$1/x$		$\arctan(3x)$	

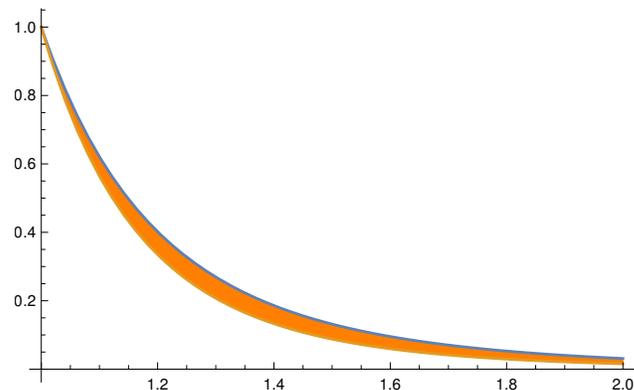
Problem 3) Matching problem (10 points) No justifications are needed.

Which of the following functions are PDF's, which are CDF's, which are neither?

Function	PDF	CDF
$\frac{1}{\sqrt{2\pi}}e^{-x^2/2}$		
$\text{Erf}(x)$		
$f(x) = e^{- x }/2$		
$f(x) = \frac{1}{\pi} \frac{1}{1+x^2}$		
$f(x) = \frac{1}{\pi} \frac{1}{\sqrt{1-x^2}}$		

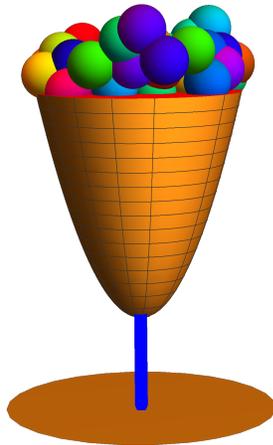
Problem 4) Area computation (10 points)

- a) (5 points) Find the area of the region enclosed by the curves $3 - x^4$ and $3x^2 - 1$.
- b) (5 points) Find the area of the region between $1/x^6$ and $1/x^5$ from $x = 1$ to $x = \infty$.



Problem 5) Volume computation (10 points)

Cody eats some magic "Bertie Botts Every Flavor Beans" from a cup which is a rotationally symmetric solid, for which the radius at position x is \sqrt{x} and $0 \leq x \leq 4$. Find the volume of Cody's candy cup.



Problem 6) Definite integrals (10 points)

Find the following definite integrals

a) (5 points) $\int_1^2 x + \tan(x) + \sin(x) + \cos(x) + \log(x) dx$.

b) (5 points) $\int_1^3 (x + 1)^3 dx$

Problem 7) Anti derivatives (10 points)

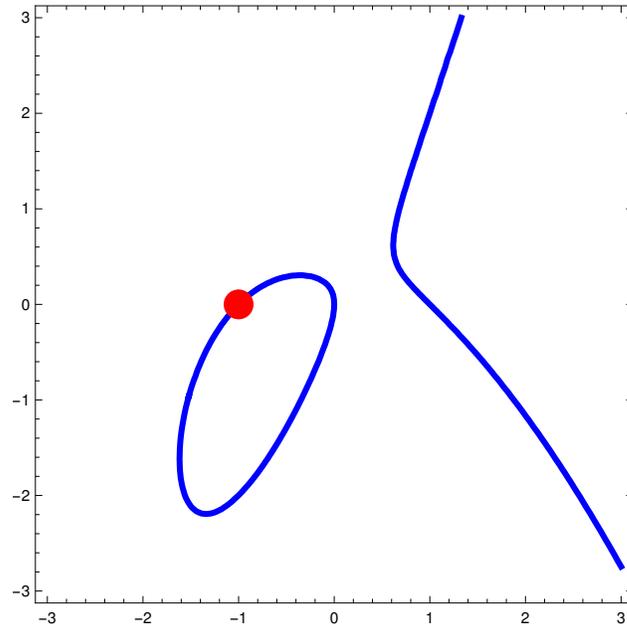
Find the following anti-derivatives

a) (5 points) $\int \sqrt{x^3} dx$

b) (5 points) $\int 4/\sqrt{x^5} dx$

Problem 8) Implicit differentiation (10 points)

The curve $y^2 = x^3 + 2xy - x$ is an example of an **elliptic curve**. Find dy/dx at the point $(-1, 0)$ without solving for y first.



Problem 9) Applications (10 points)

The probability density of the exponential distribution is given by $f(x) = (1/2)e^{-x/2}$. The probability to wait for for time x (hours) to get an idea for a good calculus exam problem is $\int_0^x f(x) dx$. What is the probability to get a good idea if we wait for $T = 10$ (hours)?

Problem 10) Applications (10 points)

What is the **average value** of the function

$$f(x) = 4 + 1/(1 + x^2)$$

on the interval $[-1, 1]$?