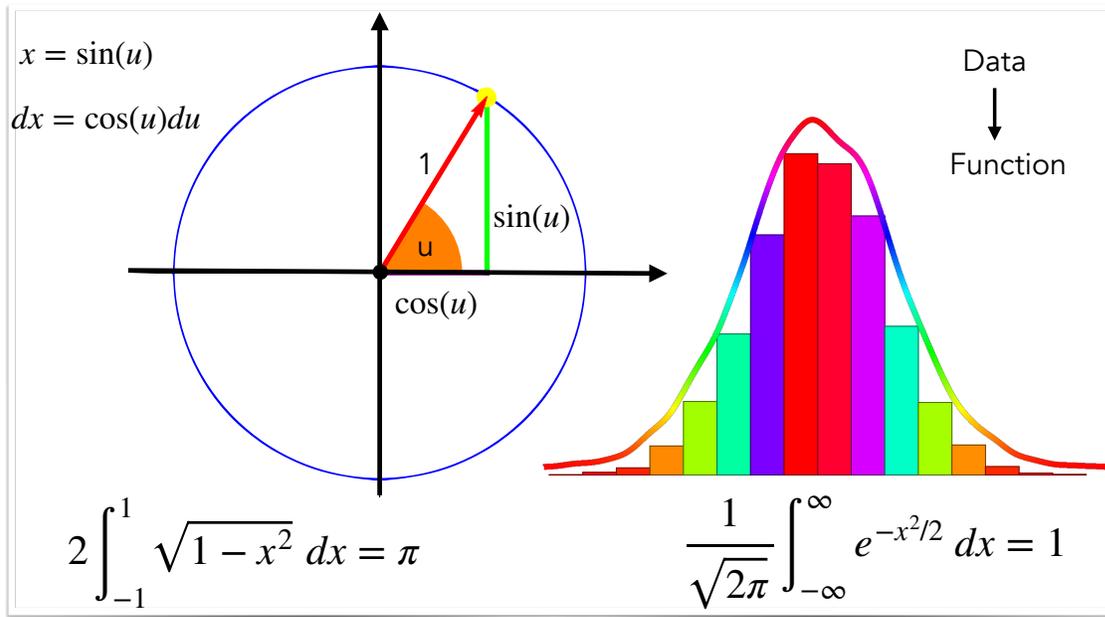


INTRODUCTION TO CALCULUS

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Content: Pages 1-149 contain 36 lectures with homework. Pages 148-156 are four data projects, pages 157-313 exam collection.

Introduction

0.1. This course is an introduction to calculus, taught during the years 2011-2014 and 2020-2021 at the Harvard college. While no previous calculus exposure is expected, basic pre-calculus skills like geometry and algebra are assumed. Even if you have seen some calculus before, a college single variable calculus course like this one will lead to a deeper, more **conceptual understanding** of the subject. It allows to see the beauty and elegance of a mathematical theory and appreciate its **applications**. While both concepts and applications are very important, also the mastery of **skills** is pivotal. Especially when doing the first steps in a new field, one has to focus on skills. Fortunately, procedures are faster to learn and teach than **insight** which requires more time as it requires **experience** and the ability to connect the dots and see similarities, patterns and being able to ask questions.

0.2. A good strategy is not to worry at first too much about reaching the ultimate picture but to focus on mastering isolated skills. The unfortunate dogma “you need first to understand!” can lead to learning blocks. Whenever one embarks into a new area of knowledge or activity, one always faces a similar challenge: at first one has to get acquainted with language and jargon, then one learns how to work out things and finally one sees patterns and gains insight and can build bridges between different already grown knowledge patches. It is important at first to just enjoy the learning while doing it. The understanding will come naturally. This is the same for all learning: if you learn to play a music instrument, a new language, to cook or to climb mountains, you first want to know the notes, the words, the ingredients and the gear. Then you learn how to play, to speak, to follow recipes or climb following a mountain guide before you start to improvise, write your own text, to create new meals or discover uncharted climbing routes.

0.3. You might wonder why it is necessary to learn a single variable theory. Isn't the world vastly **multi-dimensional**? It turns out that the **one-dimensional** point of view, the development of a single quantity over time, is extremely important. If you study the motion of the universe for example, you are interested in its expansion rate, which is a function of time and so subject to single variable calculus. If you study the spread of a virus, you are interested in the number of infected as a function of time. If you study climate or weather, you can be interested in the average global temperature over time, like decades or centuries. These are **functions of one variable**, despite the fact that the underlying mechanisms are complex systems like partial differential equations. If you are interested in finance, you might be concerned in the stock prize of a single company over time. If you probe a probability space with a random variable, then you are interested in the distribution. These are all functions of one variable again. Time is one dimensional. Single variable calculus therefore is a window to some of the deepest secrets of knowledge.

0.4. Calculus also is a large part of our scientific cultural heritage. Knowing about the historical development helps us to see today, where the difficulties are. A student of calculus essentially faces the same challenges than our ancestors who developed the field. History also produces story lines and add a dramatic element. It is only a dozen generations ago when humanity did not understood the notion of limit. In antiquity, the subject had been part of sometimes controversial philosophy. Paradoxa formulated already in antiquity illustrate the confusion which existed then. Today we have a crystal clear picture. While the ideas of calculus already trace back to the time of ancient Greece, in particular to the time of Archimedes, the subject has exploded into a **powerful tool** during last few centuries. It is now a pivotal **theoretical foundation** for other mathematical areas and scientific fields. Without exaggeration it is safe to say that calculus is one of most amazing scientific and cultural achievements of humanity.

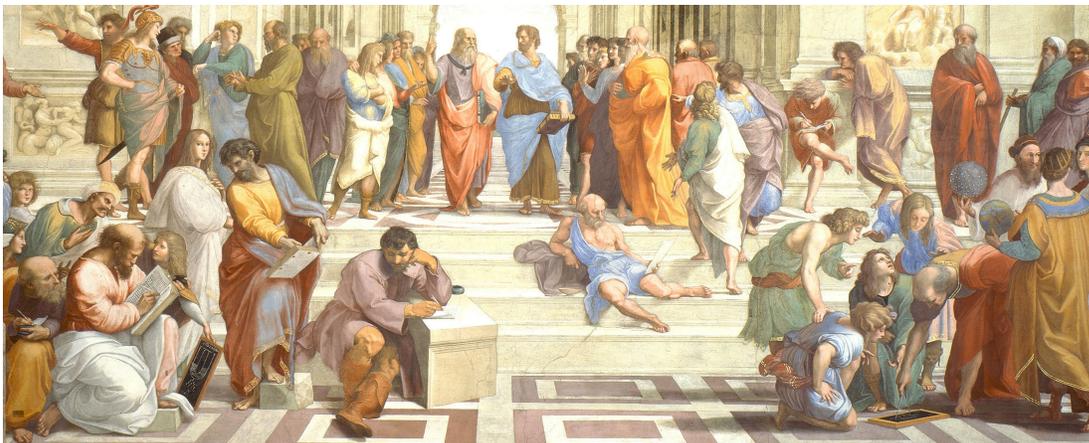
0.5. Calculus consists of differential and integral calculus. Differential calculus studies “change”, integral calculus deals with “accumulation”. The fundamental theorem of calculus links the two. The subject is very applicable to problems from other scientific disciplines. Calculus is not only important because of its content and applications like life sciences (example: tomography), data science (example: compute correlations),

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internet (example: networks), artificial intelligence (example: machine learning), geography (example: data visualization), movie and game industry (computer graphics), the ideas of calculus also enter in disguised form, in statistics, economics, computer science, in art or music theory. Do not forget that we primarily want to learn the nuts and bolts and down-to-earth techniques.

0.6. The use of computers and computer algebra systems or online tools to experiment with the mathematical structures is encouraged. The use of laptops or tablets in class to take notes is of course perfect. No computer, phone or tablet of any type will be permitted however during exams except possibly as a writing tool, when tests are taken remotely. If you get computer assistance for homework, acknowledge it in the homework. We do recommend that you work out most of the work on paper (or electronic paper). The material sticks better when you write mathematical formulas and procedures by hand. It enhances long term retention and prepares you for exams.

0.7. We do not follow any book or previous course. There are many good books. A popular text which has proven to be useful in the last decades is “Single Variable Calculus: Concepts and Contexts”, by James Stewart. Our homework problems are of similar difficulty than in such textbooks; some of them are trickier and intended to trigger discussion with other students or teaching staff. I recommend to attack each homework problem first on your own. This helps you to develop independent thinking and **problem solving skills** and prepare for the exams. You should get comfortable with the situation of being stuck at first. But even if you can solve a problem, it is helpful to discuss them with others. Mathematics is also a social activity.



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Oliver Knill, January 10th- May 10th, 2021.

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