

Unit 19

Anti derivatives

1

(A)

$$\int_0^1 e^x dx = e^x \Big|_0^1 = \boxed{e-1}$$

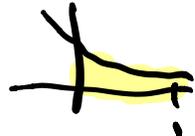
~ 1.6



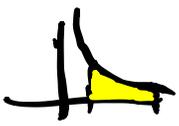
(B)

$$\int_0^1 e^{-x} dx = -e^{-x} \Big|_0^1 = \boxed{-1+e}$$

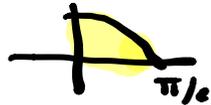

(C)

$$\int_0^1 e^{-x} dx = -e^{-x} \Big|_0^1 = \boxed{1-e^{-1}}$$


(D)

$$\int_1^2 \frac{1}{x^2} dx = -x^{-1} \Big|_1^2 = \boxed{\frac{1}{2}}$$


(E)

$$\int_0^{\pi/2} \cos x dx = 1$$


(F)

$$\int_{-2}^2 3x^4 dx = \frac{3}{5} x^5 \Big|_{-2}^2$$

$$= \frac{3 \cdot 64}{5} = \frac{192}{5}$$

2

(A) $\int \frac{1}{x^2} dx = \boxed{-\frac{1}{x} + C}$

(B) $\int \frac{1}{\cos^2 x} dx = \boxed{\tan(x) + C}$

(C) $\int x^{-2/3} dx = \boxed{x^{1/3} \cdot 3 + C}$

(D) $\int \cos x dx = \boxed{+\sin(x) + C}$

(E) $\int 4 - \frac{3}{1+x^2} dx$

$F(0) = 3$

$F(x) = 4x - 3 \arctan x + C$

what is C?

$\boxed{C = 3}$

MIT Contest Problem (AM)

Group 2

(6)

Group 6

(4)

Congratulations!

MIT contest problem

3

$$\int \frac{1}{1+e^x} dx = \text{Trick}$$

$$= \int \frac{1+e^x}{1+e^x} - \frac{e^x}{1+e^x} dx$$

$$= \int 1 - \frac{e^x}{1+e^x} dx$$

Try: $\log(1+e^x)$

$$\left(\begin{aligned} \frac{d}{dx} \log(1+e^x) \\ = \frac{1}{1+e^x} \cdot e^x \end{aligned} \right)$$

$$= x - \log(1+e^x) + C$$

4

$$\int \frac{\log x \cdot \log \log x}{x} dx$$

MIT contest prob

~~Try: $\frac{d}{dx} \log(\log(x))$~~

$$= \frac{1}{\log x} \cdot \frac{1}{x}$$

~~$\frac{d}{dx} \log(\log(\log x)) = \frac{1}{\log \log x}$~~

we are lost ...

$$\frac{1}{\log x \cdot x}$$

⑤ Integration is hard!
Sometimes impossible!

$$\int e^{-x^2} dx$$

Stats

does not have an
anti-derivative that
is elementary

we just call it

$$\text{erf}(x)$$

$$\int \frac{1}{\log x} dx$$

number
theory

we call this $\text{li}(x)$

because there is no
elementary expression.
