

# INTRODUCTION TO CALCULUS

MATH 1A

## Unit 0: Warm Up: due 1/22/2024

**Problem 1.1: Algebra:** Please simplify the following expressions

- a)  $\frac{1}{\frac{1}{3} - \frac{1}{4}}$
- b)  $(1/3)/(4/9)$
- c)  $\sqrt{x^6}x/x^{-2}$
- d)  $((2^4)^3) - (2^3)^4$
- e)  $\ln(e^{10}e^{11})$ .

**Solution:**

- a) 12
- b)  $3/4$
- c)  $x^6$
- d) 0
- e) 21

**Problem 1.2: Equations:** Please solve the following equations for  $x$ .

- a)  $x^2 + 1 = 2x$
- b)  $\sin(x) = 1/2$
- c)  $2^x = 1$
- d)  $\tan(x) = \sin(x)$
- e)  $\sqrt{x} + x = x\sqrt{x}$ .

**Solution:**

- a)  $x = 1$ .
- b)  $\pi/6 = 30^\circ$ .
- c)  $x = 0$ .
- d)  $\sin(x) = 0$  or  $\cos(x) = 1$  which means  $x = k\pi$  with integer  $k$ .
- e)  $(\sqrt{5} + 3)/2$ .

**Problem 1.3: Graphing:** Graph the following functions

- a)  $f(x) = 3x + 2$
- b)  $f(x) = 2^x - 5$
- c)  $f(x) = x^3 - x$
- d)  $f(x) = e^{-x^2}$
- e)  $f(x) = \sin(2x) + 1$

**Solution:**

For this problem you can plot it by hand, once you know what happens. Good things to look for are how the function grows, especially in the long term.

- a) This is a line of slope 3 passing through (0,2)
- b) This is an exponential function which monotonically increases goes asymptotically to -4 for  $x \rightarrow -\infty$  and to  $\infty$  for  $x \rightarrow \infty$ .
- c) This is a cubic function which comes from -infinity, goes to a max, then a min and then goes to infinity.
- d) this is the gaussian (bell curve). It is never zero, never negative and goes to zero on both ends. e) this is a wave. It is never negative but has roots when  $\sin(2x) = -1$  which is at  $3\pi/4 \pm k\pi/2$ .

**Problem 1.4: Geometry:** A triangle has side lengths 3, 4, 5.

- a) What is the largest angle in the triangle?
- b) What is the area of the triangle?
- c) What is  $\arcsin(4/5)$ ?
- d) Solve the equation  $\arctan(3/4) = \arcsin(x)$  for  $x$ .
- e) What is  $\arccos(3/5) + \arcsin(3/5)$ ?

**Solution:**

- a)  $90^\circ$
- b)  $3 * 4/2 = 6$
- c)  $12/3 + 12/4 + 12/5 = 9.4$
- d) Use the triangle  $\arctan(3/4) = y$  is an angle if this is  $\arcsin(3/5)$  then  $\sin(y) = 3/5 = 0.6$ .
- e) The sum of the two smaller angles of the triangle is  $\pi/2 = 90$  degrees.

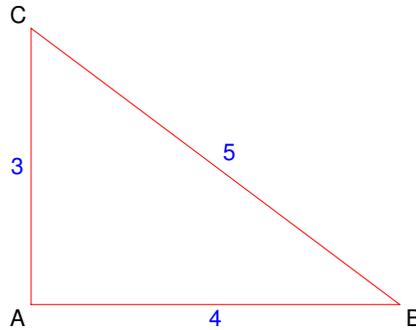


FIGURE 1. The 3-4-5 triangle.

**Problem 1.5: Exponentials:** Which of the following statements are true?

if  $x + y = z$ ?

- a)  $2^x + 2^y = 2^z$
- b)  $2^x 2^y = 2^z$
- c)  $2^{x+y} = 2^z$
- d)  $x^2 + y^2 = z^2$
- e)  $\log_2(x) + \log_2(y) = \log_2(z)$

**Solution:**

- a) False
- b) True
- c) True d) False e) False

**Problem 1.6: Modeling:** You count 1000 bacteria of a colony today  $t = 0$  and 32000 after  $t = 5$  days. How many bacteria are there in 9 days?

**Solution:**

$10002^{kt} = f(t)$ . We see that  $k = 1$ . So, in 9 days there are  $1000 \cdot 2^9$  bacteria.

**Problem 1.7: Laws:** a) Write down 3 laws for exponentiation.  
b) Write down 3 laws for logarithms.

**Solution:**

a)  $a^x * b^x = (ab)^x$

$(a^x)^y = a^{xy}$

$a^0 = 1$

b)  $\log(xy) = \log(x) + \log(y)$

$\log(a^x) = x \log(a)$

$\log(1) = 0$

**Problem 1.8: Definitions:** Summarize the definitions for  $\sin(x)$ ,  $\cos(x)$ ,  $\tan(x)$  and  $\cot(x)$ .

**Solution:**

SOH-CAH-TOA-CAO

**Problem 1.9: Triangles:** There are 2 important special triangles for which all angles are known explicitly. What are these angles. (You should find 4).

**Solution:**

45,30,60,90

**Problem 1.10:** Please check the syllabus if necessary:

- What are the names of your instructors and CA's
- Summarize the homework policies.
- Is Chat GPT allowed in this course?
- When are the midterm dates?
- State the first sentence of the Harvard honor code.

**Solution:**

- a) Oliver, Eugenia, Jota and Jacobo
- b) Due every class. No late HW policy, least 3 PS scores are deleted.
- c) Not for HW and exams.
- d) Feb 28 and April 3.
- e) "Members of the Harvard College community commit themselves to producing academic work of integrity."

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