

INTRODUCTION TO CALCULUS

MATH 1A

UNIT 8: WORKSHEET

Problem 1: Get the slope of the tangent to the graph of $f(x) = xe^{-x}$ at $x = 0$.

Problem 2: We know that $\frac{d}{dx}x^4$ is $4x^3$. (Was done in class the "old way" using a limit.) Derive this here from the product rule for $x^2 \cdot x^2$.

Problem 3: Find the derivative of $1/x^3$ using the quotient rule.

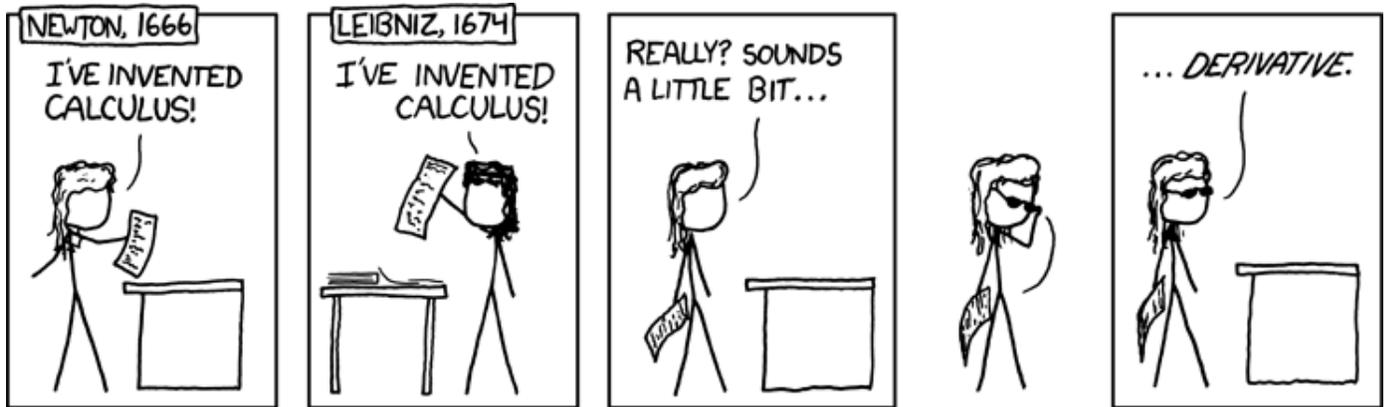
Problem 4: Find the derivative of the function $\cos(x)x$ at $x = 0$.

Problem 5: If $f(x) = \sqrt{x}/x$, what is $f'(x)$? What is $f'(1)$?

Problem 6: Find the derivative of $1/e^x$ at $x = 1$.

Problem 7: Remember the formula $\sin(2x) = 2 \sin(x) \cos(x)$? Differentiate both sides to get a formula for $\cos(2x)$.

Problem 8: You are given $\arctan'(x) = 1/(1 + x^2)$. Find $\arctan''(x)$.



Source: XKCD

I.

NOVA METHODUS PRO MAXIMIS ET MINIMIS, ITEMQUE TANGENTIBUS, QUAE NEC FRACTAS NEC IRRATIONALES QUANTITATES MORATUR, ET SINGULARE PRO ILLIS CALCULI GENUS*).

Sit (fig. 111) axis AX , et curvae plures, ut VV, WW, YY, ZZ , quarum ordinatae ad axem normales, VX, WX, YX, ZX , quae vocentur respective v, w, y, x , et ipsa AX , abscissa ab axe, vocetur x . Tangentes sint VB, WC, YD, ZE , axi occurrentes respective in punctis B, C, D, E . Jam recta aliqua pro arbitrio assumpta vocetur dx , et recta, quae sit ad dx , ut v (vel w , vel y , vel z) est ad XB (vel XC , vel XD , vel XE) vocetur dv (vel dw , vel dy , vel dz) sive differentia ipsarum v (vel ipsarum w , vel y , vel z). His positus, calculi regulae erunt tales.

Sit a quantitas data constans, erit da aequalis 0 , et dax erit aequalis adx . Si sit y aequ. v (seu ordinata quaevis curvae YY aequalis cuius ordinatae respondentis curvae VV) erit dy aequ. dv . Jam *Additio et Subtractio*: si sit $z = y + w + x$ aequ. v , erit $dz = dy + dw + dx$ seu dv aequ. $dz = dy + dw + dx$. *Multiplicatio*: $d\sqrt{xy}$ aequ. $x dv + v dx$, seu posito y aequ. xv , fiet dy aequ. $x dv + v dx$. In arbitrio enim est vel formulam, ut xv , vel compendio pro ea literam, ut y , adhibere. Notandum, et x et dx eodem modo in hoc calculo tractari, ut y et dy , vel aliam literam indeterminatam cum sua differentiali. Notandum etiam, non dari semper regressum a differentiali Aequatione, nisi cum quadam cautione, de quo alibi.

Porro *Divisio*: $d\frac{v}{y}$ vel (posito z aequ. $\frac{v}{y}$) dz aequ. $\frac{\pm v dy \mp y dv}{yy}$.

Quoad *Signa* hoc probe notandum, cum in calculo pro litera substituitur simpliciter ejus differentialis, servari quidem eadem signa, et pro $+z$ scribi $+dz$, pro $-z$ scribi $-dz$, ut ex addi-

*) Act. Erud. Lips. an. 1684.

Leibniz 1684 paper in which the product and quotient rule is introduced.