INTRODUCTION TO CALCULUS

MATH 1A

UNIT 9: WORKSHEET

Problem 1: Compute the limit:

$$\lim_{x\to 0} \frac{\sin(3x) + x}{x^2 + x}$$

Solution:
4

Problem 2: Evaluate the following limit:

$$\lim_{x\to 1}(x - 1)/\log(x - 1)$$

Solution:
L'Hospital gives 0. We should check however, whether this is really a case for Hospital. The example is not appropriate because it is not of the form 0/0. It is of the form 0/infinity which is zero.

Problem 3: Find the limit:

$$\lim_{x\to\infty}\log(7x + 3)/\log(9x + 4)$$

Solution:
Hospital and then again applying Hospital gives 1.

Problem 4: Find the limit (see Figure 1 below)
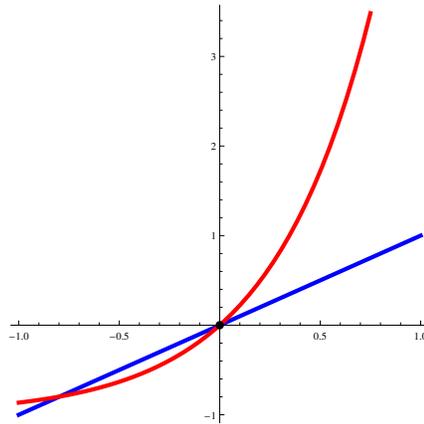
$$\lim_{x\to 0}\frac{e^{2x} - 1}{x} \ ?$$

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Solution:
The limit is $1/2$.

Problem 5: And now evaluate the following limit

$$\lim_{x \to -1} (x^2 + 2x + 1)/(x + 1) \,.$$

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Solution:
Healing simplifies to $x + 1$ which has the limit 0. Also sending it to the hospital works. It is the limit $(2x + 2)/1$ for $x \to -1$ which is 0.
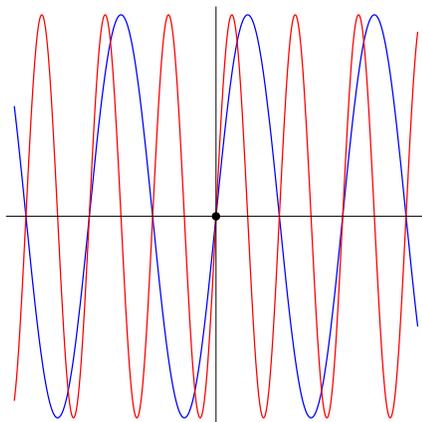

The functions $e^{2x} - 1$ and x.

Problem 6: Apply l'Hospital's rule to get the limit of

$$f(x) = \frac{\sin(200x)}{\sin(300x)}$$

for $x \to 0$.

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Solution:
$200/300$.

Problem 7:   What does l'Hospital's rule tell about the limit

$$f(x) = \frac{\sin(200x)}{\sin(300x)}$$

for $x \to \pi$.

Solution:
We again gives $200/300$ Evaluating $\cos(200\pi)$ gives 1.

Problem 8:   What does l'Hospital's rule tell about the limit

$$f(x) = \frac{\cos(200x)}{\cos(300x)}$$

for $x \to 0$.

Solution:
Nothing. We can evaluate the limit directly and see that the result is 1.

Problem 9:   What does l'Hospital tell you about

$$\lim_{x \to 1}(x + 1)/(x - 1)$$

Solution:
This is a case 2/0 which is infinity. There is no limit. Hospital can not be applied. The patient is dead already, can not be rescued.

Problem 10: And finally a joke:

$$f(x) = \lim_{x \to 0} 1/1 =_{\text{Hospital}} \lim_{x \to 0} 0/0 = \textbf{not defined}$$

Solution:
1/1 is perfectly healthy and does not have to be sent to the hospital! In this case actually the treatement is deadly.

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