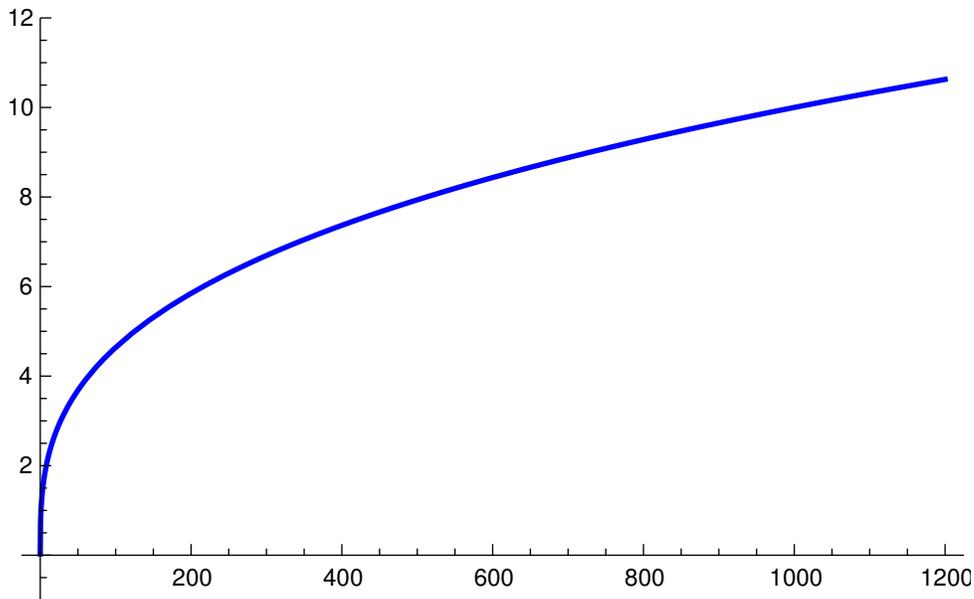


INTRODUCTION TO CALCULUS

MATH 1A

UNIT 11: WORKSHEET

Problem 1: The following graph shows the cube root function $f(x) = x^{1/3}$. Locate the point $a = 512$, its value $f(a)$ and draw the linearization of f at a . How close to you estimate the error $f(600) - L(600)$ just by looking at the graphs? Can you compute the difference explicitly?



Solution:

When looking at the picture, $f(600) - L(600)$ is small, smaller than $1/10$. The linearization is $L(x) = 8 + (x - 512)/(3 * 64)$ which gives 8.46 The actual number is 8.434 which is maybe 0.03 off.

Problem 2: Estimate $\sqrt{150}$ using linear approximation at $a = 144$?

Solution:

$$12 + 6/24 = 12.25$$

Problem 3: Estimate $\ln(2)$ using linear approximation at $a = 1$.

Solution:

We have $L(x) = \ln(1) + (1/1)(x - 1)$ which gives the value 1. $\ln(2)$ is smaller 0.65. here the linear approximation is not that good. But 2 is very far off from 1.

Problem 4: Estimate $\sin(0.1)$ using linear approximation.

Solution:

$L(x) = 0 + 1(x - 0) = x$. The estimate is 1.

Problem 5: Estimate $\arctan(1.1)$ using linear approximation at $a = 1.0$.

Solution:

We have to compute $\arctan(1) = \pi/4$ and $\arctan'(1) = 1/2$. The linearization is $\pi/4 + 1/2(x - 1)$. We get $\pi/4 + 0.1/2$

Problem 6: Estimate $\tan(0.3)$ using linear approximation at $a = 0$.

Solution:

$\tan(0) = 0, \tan'(0) = 1$. The linear approximation function is $L(x) = x$. The estimate is 0.3.